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PRACTICAL MECHANICS

AND

ALLIED SUBJECTS

BY

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PREFACE

This book together with that on "Practical Applied Mathematics" by the same author embodies the results of five years' experience in the organization and development of a system of railroad shop apprentice schools, as well as extended investigations of the work of public, private, trade, vocational and corporation schools of manufacturing industries and railroads in this country and abroad.

In the presentation no formal distinction is made of the various branches of elementary mathematics and science involved. The subject matter is presented in the two volumes under the general titles of "Practical Applied Mathematics" and "Practical Mechanics and Allied Subjects," although a careful and logical arrangement and sequence have been followed in the presentation.

In organizing trade apprentice schools difficulty has been experienced in securing a text practically adapted to the needs of these schools. This difficulty has led the author to develop a series of instruction sheets which are believed to be thoroughly practical. This volume with that on "Practical Applied Mathematics" is essentially a presentation of problems arising in shop experience in the mechanical trades. It shows the use of mathematics as a tool.

The material included in this work has been made sufficiently broad to apply not only to railroad schools but it is believed to schools in the mechanical trades generally. It is also felt that this work may be used to advantage in the public schools in technical branches, as well as in trade and vocational schools, either as a regular or as a supplementary text.

The material for each book is presented in twenty chapters, each dealing with a definite subject. This affords a flexibility highly desirable in assigning the work and enables it to be given entirely or in part according to specific needs.

In preparing this work the author acknowledges indebtedness to Pennsylvania Railroad Motive Power The author is indebted to Dr. J. P. Officials at Altoona. Jackson, State Commissioner of Labor and Industry of Pennsylvania, Professor J. A. Moyer, Head of the Department of Mechanical Engineering and Director of the Engineering Extension Division of the Pennsylvania State College and Mr. M. B. King, State Expert Assistant in Industrial Education of Pennsylvania, for reading the proofs and offering valuable criticisms and suggestions. He is especially indebted to Professor J. H. Yoder and Mr. E. W. Hughes of the School of Engineering of the Pennsylvania State College, detailed as Instructors in the Pennsylvania Railroad System of Apprentice Schools. They have assisted in the preparation of the manuscript, carefully read the proofs and offered many valuable suggestions.

JOSEPH W. L. HALE.

ALTOONA, PA. June 10, 1915

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PRACTICAL MECHANICS AND ALLIED SUBJECTS

CHAPTER I

FORCES

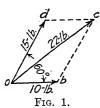
1. Definitions.—In our every day work we deal with forces of one kind or another. A pressure is a force. The earth exerts a force of attraction for all bodies or objects on its surface. Steam exerts a force or pressure in a locomotive boiler and cylinder. In grinding tools a force of friction has to be overcome. The wind exerts a force or pressure against the surface which it strikes.

When forces act on objects they move the objects if the forces are great enough to overcome the resistance to motion, and even if the objects do not move the forces produce a *stress* and *pressure* within and on the objects.

Forces are measured in pounds and tons. When we consider forces acting on objects we must know how the forces are applied, the direction of the forces and their values in pounds and tons. In working out problems forces are often represented by straight lines. The length of the line represents to scale the value of the force as 100 lb., 20 tons, etc., and the direction of the line, the direction in which the force acts as vertical, horizontal, etc.

2. Forces Passing through a Point.—In Fig. 1 the line Od 1 in. in length represents a force of 15 lb. and the line Ob 2/3 of an in. in length represents a force of 10 lb. Also force Ob is horizontal and force Od acts at

an angle of 60° with the horizontal.

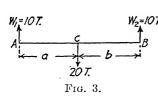


Od and bc.

Two or more forces may be combined by the "parallelogram" method giving a single force having the same effect as the separate forces acting together, as for example in Fig. 1 forces of 10 lb. and 15 lb. acting on an object at 0 and 60° apart may be replaced by

a single force Oc of 22 lb. which is the same in effect as the forces of 10 lb. and 15 lb. acting together. The ingle force Oc is called the "resultant" of the forces Od and Oc are parallel and equal, and the same is true of lines Oc

Rule.—The resultant of a number of forces is the direct sum of these forces only when they act in the same straight line, as in Fig. 2 and in the same direction.



Any force has the greatest Wz=107. effect in the direction in which it acts, and no effect in a direction at right angles to that in which it acts.

3. Parallel Forces.—Parallel forces are those which act

in directions parallel to one another and which do not meet. In Fig. 3 is shown three forces, one of 20 tons and two of 10 tons each, balancing the *load* of 20 tons. For parallel forces as in this Fig. $W_1 + W_2 = 20$, and

$$\frac{W_1}{20} = \frac{b}{a+b}$$

and

$$\frac{W_2}{20} = \frac{a}{a+b}$$

PROBLEMS

1. What is the total force or pressure acting on a 14-in. piston in a steam cylinder with a steam pressure of 180 lb. per sq. in.?

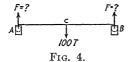
2. What must be the diameter of a piston in an air cylinder for a total pressure of 3000 lb. with a pressure per sq. in. of 60 lb.?

3. If the total pressure produced on a piston by steam in an 18-in. cylinder is 45,600 lb., what is the steam pressure in lb. per sq. in.?

4. What force of attraction does the earth produce on a steel rod 4 in. in diam, and 3 ft. 6 in. in length?

5. Two forces of 20 lb. and 30 lb. act on an object in opposite directions. What is the "resultant" force? If these forces acted at right angles to one another what would be the "resultant" force?

6. What is the total pressure produced on the supporting structure for a water tank if the tank is 12 ft. high and 8 ft. inside



diam. and full of water? The material of the tank weighs 1280 lb.

7. A traveling crane (Fig. 4) lifts a 100-ton locomotive. With the load half way between the crane supports A and B, what pressure is brought on each of these supports? How is this pressure changed when the load c moves nearer end A?

8. Using Fig. 4, find the pressures produced on supports at both A and B with the following loads and distances of the loads from A and B.

	Load at C	Distance AC	$\begin{array}{c} \textbf{Distance} \\ BC \end{array}$	Pressure at A	Pressure at B
1	20 tons	20 ft.	40 ft.		
2	15 tons	32 ft.	28 ft.		
3	100 tons	10 ft.	50 ft.		
4	2500 lb.	50 ft.	10 ft.		
5	30 tons	42 ft.	18 ft.		
6	390 lb.	25 ft.	35 ft.		

9. What steam pressure is used on locomotives? Is the same pressure used for heating cars and the shops?

Give complete answer with reasons.

10. If the steam pressure is 30 lb. per sq. in., what is the total force tending to blow the end out of a steam pipe 4 in. inside diam.?

CHAPTER II

GRAVITATION. CENTER OF GRAVITY

4. Definition of Gravity.—The attractive force which every object in the universe has on every other object is called "gravitation" and this depends on the quantity of material composing the objects.

The force of attraction of the earth for objects is called the force of gravity and is what gives objects their "weight." A spring balance in giving the "weight" of any object measures merely the force of attraction of the earth for that object.

On account of "the force of gravity" which tends to pull objects toward the center of the earth work has to be done in lifting them and for the same reason elevated objects have power to do work on account of their elevated position, as in the case of water above a fall, elevated weights, train on a down grade, lifted hammer, etc. A locomotive hauling a train up a grade consumes more coal than when the train is on level track since work has to be done in lifting the weight of the train against the force of gravity as well as overcoming frictional resistances. A train descending a grade has work done on it by the force of gravity. The train will coast and often attain a high speed without the use of any steam.

5. Gravity Action On Freely Falling Objects. The "force of gravity" acting on objects free to fall increases their speed about 32.2 ft. per sec. each second of the

- fall. For example, a weight starting from rest and falling freely will have a speed of 96.6 ft. per sec. at the end of the third second of its fall and 161.0 ft. per sec. at the end of the fifth second of its fall.
- 6. Work Done Against Gravity.—The work done in lifting objects against "the force of gravity" is measured in *foot-pounds* and is equal to the weight of the object in pounds multiplied by the vertical height in feet through which the object is lifted.

PROBLEMS

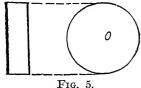
- 1. A train is made up of a locomotive weighing 272,000 lb., a 158,000-lb. tender, a baggage and mail car weighing 86,000 lb. and 5 Pullman sleepers weighing together 625,000 lb. How much work is required to lift the train up a mile of grade rising 1 ft. in 250 ft.?
- 2. How much work in foot-pounds can a 2-ton hammer do in each fall if it is raised 16 ft.?
- 3. How much work in foot-pounds is required to load the following lot of tires on a flat car, if the average distance through which they are raised is 6 ft.?

50 tires each weighing 452 lb. 37 tires each weighing 643 lb. 24 tires each weighing 598 lb.

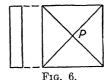
- 16 tires each weighing 544 lb.
- **4.** If a 10-ton casting is raised 23 ft. by a traveling crane, how much work is done? If one horsepower is the performance of 33,000 ft.-lb. of work per min. and the above casting is raised in 1 min., what horsepower does the motor on the crane exert?
- 5. Using the definition for "horsepower" given in Prob. 4; what work in foot-pounds could a 10-h.p. engine do in 5 min. if it ran at full capacity for this time?
- **6.** A single acting pump raises 1600 gal. of water every minute through a distance of 5 ft. How much work against gravity does the pump do per hr.?
 - 7. A train weighing 1000 tons descends 2 miles of grade dropping

1 ft. in 200 ft. What work in foot-pounds does "gravity" do on the train in the descent?

- 8. How much work is done per hr. in raising a 1-ton hammer through a vertical height of 4 ft. an average of fifteen times per min.?
- 9. If an object falls freely for 7 sec., what is its speed at the end of this time? Would a bullet and a feather fall side by side in the air? Has air friction any effect on the speed of a falling object?
- 7. Definition of Center of Gravity.—In the fourth paragraph we learned that the earth attracts toward its center every particle of every object on its surface, and this force of attraction called the force of gravity gives objects their weight. When solving problems in mechan-



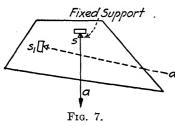




es it, is often necessary to find a point in an object at which its entire weight may be considered as concenrated. This point about which an object would balnnce is called its center of gravity. For example, a circular steel plate as in Fig. 5, has its "center of gravity" at he center 0, half way between the faces of the plate. If he metal in this plate weighed .30 lb per cu. in. and the plate contained 100 cu. in. of metal we could consider ts entire weight of $.30 \times 100$ or 30 lb. as acting at 0, ustead of taking the weight of each cu. in. separately. This gives only one value, 30 to work with instead of 100 values, each of .30.

The entire weight of a square uniform metal plate as in Fig. 6 may be taken at a point P at the intersection of the diagonals of the square and half way between the faces of the plate. The center of gravity of a cylinder is at a point on the axis of the cylinder and half way between the upper and lower bases.

The position of the center of gravity of an object such as a metal sheet, as shown in Fig. 7, can be found experimentally by using a plumb line and a support on which the object can move freely. When an object is suspended from a support on which it can move freely the



"center of weight" or "center of gravity" is brought as low as possible, and therefore lies in a vertical line drawn through the point of support. Using this principle an object as a steel plate shown in the figure is suspended freely from

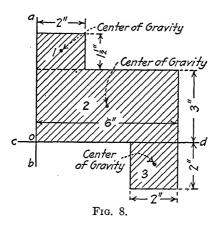
the support "s" and a plumb line dropped from the point of support as shown. The direction of the line is then marked on the plate giving line "sa."

The plate is next supported from a second point and with the plumb line we get line s_1a_1 . The point where lines sa and s_1a_1 cross marks the position of the "center of gravity" of the plate, half way between the front and back faces, if the plate is of uniform thickness throughout.

The center of gravity of sections made up of a number of simple sections as shown in Fig. 8, composed in this case of three rectangles, is found as follows: First we choose two "reference axes" as lines ab and cd crossing at 0. Next we find the product of the area of each simple

figure and the distance of *its* center of gravity from the chosen axis, then we add these products together and divide their sum by the sum of the areas of the simple figures.

For example, in the figure shown, the area of rectangle number 1 is $2 \times 1\frac{1}{2}$ or 3 sq. in. The area of No. 2 is 6×3 or 18 sq. in. and of No. 3, 2×2 or 4 sq. in. The



sum of the products of these areas by the distances of their respective centers of gravity from axis ab gives $3 \times 1 + 18 \times 3 + 4 \times 5$ or 77. Dividing this by the sum of the areas of the simple figures we have

$$\frac{77}{3+18+4}$$
 or $\frac{77}{25}$ or 3.08 in.

which is the distance of the center of gravity of the whole figure from line or axis ab.

Working in the same way from axis cd we have the equation

Distance of c.g. from line

$$cd = \frac{3 \times 3.75 + 18 \times 1.5 - 4 \times 1}{3 + 18 + 4}$$

or $\frac{34.25}{25}$ which equals 1.37 in.

The point where the lines cross as indicated in Fig. 9 is the position of the center of gravity of the figure or the point on which the whole plate would balance. In the equation above the last term in the numerator that

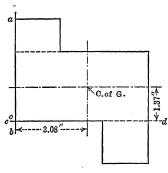


Fig. 9.

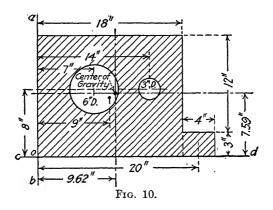
is 4×1 has to be subtracted as shown rather than added since the area 4 sq. in. is on the *other side* of the reference axis cd, and if this was, for example, a metal plate we were calculating, the weight represented by the lower rectangle would help counterbalance the weight of the parts on the other side of the axis cd.

Example.—Find the center of gravity of the metal plate shown in Fig. 10 in which are cut two holes 3 in and 6 in, in diam.

Solution.—Distance of center of gravity from axis ab equals

$$\frac{18 \times 15 \times 9 + 4 \times 3 \times 20 - 6^2 \times .785 \times 7 - 3^2 \times .785 \times 14}{18 \times 15 + 4 \times 3 - 6^2 \times .785 - 3^2 \times .785}$$

or $\frac{2373.3}{246.68}$ which equals 9.62 in.



Distance of center of gravity from axis cd equals

$$\frac{18 \times 15 \times 7.5 + 4 \times 3 \times 1.5 - 6^2 \times .785 \times 8 - 3^2 \times .785 \times 8}{18 \times 15 + 4 \times 3 - 6^2 \times .785 - 3^2 \times .785}$$

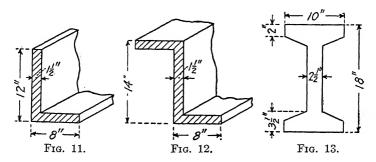
or $\frac{1873.4}{246.68}$ which equals 7.59 in.

PROBLEMS

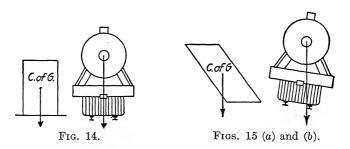
- 10. Find the center of gravity of the angle iron section shown in Fig. 11.
- 11. Find the center of gravity of the "Z" beam section shown in Fig. 12. If this beam is 10 ft. long, find its weight at .28 lb. per cu. in. for steel.
 - 12. Find the center of gravity of the I-beam section shown in

Fig. 13, and its weight if 18 ft. long and steel weighs .28 lb. per cu. in.

8. Stability of Objects.—Whether an object will stand up or fall over, that is, its "stability," depends upon



the position of its "center of gravity" with respect to its base. The rule is as follows: If a *vertical* line through the "center of gravity" passes inside the base of an object as in Fig. 14, the object will not fall, but if the



vertical line comes outside the base as in Fig. 15 α , the object will fall. From this it follows that objects which are heavy and have a large base are less likely to fall, than those which are lighter and have a smaller base,

and also the *lower* the "center of gravity" in an object the less likely it is to fall. The center of gravity of an "area" or "surface" of any kind is understood to mean that point about which a very thin sheet representing the surface would balance. Fig. 15b representing the end of a locomotive shows the change in position of the "center of gravity" with respect to the base when the locomotive is on a depressed rail on a curve.

PROBLEMS

- 13. A steel plate 2 ft. × 4 ft. is % in. thick. What is its weight and at what point in the plate can this entire weight be considered as concentrated? What is this point called?
 - 14. Where is the "center of gravity" of a sphere located?
- 15. At what point in a steel bar 4 in. in diam. and 6 ft. long can the entire weight be considered as acting? What is this weight?
- 16. Where is the center of gravity of a cube located? Find the weight and center of gravity of a cube of steel 2 in. on a side.
- 17. Why is it desirable to have the center of gravity of a pulley in line with the center line of the shaft on which the pulley works?
- 18. How could you find the center of gravity of any triangularly shaped piece of steel plate?
- 19. Is it necessary to use a counterbalance on a fly-wheel, and if so explain why?

CHAPTER III

DENSITY AND SPECIFIC GRAVITY

9. Definitions and Problems.—The density of an object means its mass or quantity of matter per unit of volume. Suppose a cubic foot of metal weighs 450 lb., we may then say that the density of the metal is 450 lb. per cu. ft. Since all objects are not of the same weight it is found convenient to have a ready means for calculating their relative weights. In order to do this, tables are prepared which give us the weights of unit volumes of materials compared with the weight of the same volume of some material taken as a standard.

Specific Gravity is the ratio of the density of two objects. For example: If a cubic foot of metal weighs 490 lb, and it is compared with the weight of a cubic foot of water which is about 62.5 lb,, the ratio is $\frac{490}{62.5}$ or 7.83 which is the specific gravity of the metal referred to water.

If a block of granite containing 1 cu. ft. volume weighs 162 lb., its weight compared to a cubic foot of water is $\frac{162}{62.5}$ or 2.59 which is its specific gravity referred to water as a standard. In the same way we can compare the weights of other materials with water and prepare a table which we can use to advantage in calculations involving the weights of the materials most u ed.

In any standard handbook may be found such tables

already prepared for a large number of substances. In making out these tables the weights of the bodies if they are solid or liquid are compared to the weight of the same volume of water as a standard, the weight of the water being taken as that at its temperature of greatest weight, namely, 4° Cent. If we are finding the relative weight of a gas we use hydrogen gas as a standard.

The specific gravity of a substance is therefore the ratio of the weight of a given volume of the substance to the weight of a similar volume of a substance taken as a standard. The specific gravity of a substance is therefore merely a number or a ratio and has no name attached to it.

If W is the weight of a substance, S the weight of the same volume of a standard substance and g the specific gravity of the substance we have the rule $g = \frac{W}{S}$. From this rule $W = g \times S$, that is, the weight of the substance is equal to its specific gravity times the weight of the same volume of the standard substance. Also we can write the equation in the form $S = \frac{W}{g}$.

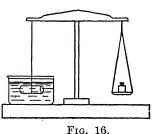
It is known that an object entirely immersed in a liquid loses in weight an amount equal to the weight of the volume of the liquid displaced by the object immersed. This principle enables us to determine easily the specific gravity of some substances by completely immersing them in liquids in which they are not soluble.

Example.—As shown in Fig. 16, a small cast-iron cylinder weighs in air 1.12 lb. When completely immersed in water as shown its weight is only .965 lb. The loss in weight of the object due to submerging it in

water is therefore .155 lb. and its specific gravity equals

 $\frac{\text{weight in air}}{\text{loss of weight when immersed in water}} = \frac{1.12 \text{ lb.}}{.155 \text{ lb.}} \text{ or } 7.22.$

Evidently we could not use this method with water as



a liquid if we wished to find the specific gravity of sugar, since sugar is soluble in water. In the case of substances which are dissolved by water we may use other liquids as the standard by which the substance is not dissolved and compare the weight of this new liquid with the weight

of water in determining the specific gravity.

Below is given a table of the approximate specific gravities of some of the most common materials.

APPROXIMATE SPECIFIC GRAVITIES

Iron, cast	7.21	Copper	8.79	Granite	2.6
Iron, wrought	7.78	Nickel	8.8	Glass	2.9
Steel	7.92	Mercury, at 60	13.6	Pine, white	. 55
Lead	11.3	Gold	19.26	Pine, yellow .	.66
Tin	7.29	Silver	10.47	Oak, white	.77
Zinc	7.19	Aluminum	2.67	Marble	2.7
Brass	7.82	Cork	.24	Ice	.917

Example.—Using the above table suppose we wish to find the weight of an iron casting containing $2\frac{1}{2}$ cu. ft.

Solution.—One cubic foot of water weighs 62.5 lb. The specific gravity of cast iron is 7.21, that is, the cast iron is 7.21 times heavier than water, therefore the weight of the casting equals

Example.—What is the weight of a white-oak beam $4 \text{ in.} \times 8 \text{ in.}$ and 18 ft. long?

Solution.—We first find the volume of the beam in cubic feet which is equal to $\frac{4}{12} \times \frac{8}{12} \times 18$ or 4.0 cu. ft.

Since from the table we see that the specific gravity of white oak is .77 that is, since it is .77 times as heavy as water the weight of the beam equals

cubic feet
$$\times$$
 62.5 \times .77

that is $4.0 \times 62.5 \times .77$ or 192.5 lb.

Example.—A timber is thrown into still water and it is found that it floats one-third out of water, what is its specific gravity?

Solution.—Since the timber displaces a volume of water equal to $\frac{2}{3}$ of its own volume it is held up or buoyed up by a force equal to the weight of a volume of water $\frac{2}{3}$ that of the volume of the timber. The weight of the timber is therefore only $\frac{2}{3}$ of the weight of the same volume of water, that is, its specific gravity is .67.

Example.—What is the volume of 100 lb. of copper?

Solution.—One cubic inch of water weighs $\frac{62.5}{1728}$ or .0361 lb. Since the specific gravity of copper is 8.79, 1 cu. in. of copper weighs $8.79 \times .0361$ or .317 lb. There are therefore as many cubic inches of copper in 100 lb. as .317 lb. the weight of 1 cu. in. of copper is contained into the total weight of copper or $\frac{100}{.317} = 315.4$ cu. in.

PROBLEMS

1. Using the table of specific gravities, find the weight of 8 cu. ft. of wrought iron.

2. Find the weight of a white pine timber 4 in. is 6 in. and 14 ft. long.

3. How many pounds heavier is a cubic foot of lead than a

cubic foot of wrought iron?

4. Find the number of cubic feet in a marble block weighing 21/2 tons.

5. A cylindrical casting 2½ in, in diam, and 8 in, long weighs

11.0 lb. What is its specific gravity?

- 6. A tank car has a capacity of 50,000 lb. If it is to be filled with petroleum having a specific gravity of .832, how many cubic feet and how many gallons can the tank hold?
- 7. The pressure of the atmosphere per square inch is about 14.7 lb. How high a column of mercury 1 sq. in, in cross section is required to produce this pressure?

8. A steel casting displaces 4.84 gal. of water. What is its

weight?

9. Two hundred cubic inches of brass are made up of 150 parts of copper and 50 parts of zine by volume. Find its specific gravity. Solution:

The weight of 1 cu. in. of water = .0361 lb.

The weight of 1 cu, in, of copper = $8.79 \times .0361$ or .317 lb. Therefore 150 cu, in, weigh 47.6 lb. The weight of 1 cu, in, of zinc = $7.19 \times .0361$ or .260 lb. Therefore 50 cu, in weigh 13.0 lb.

If S is the specific gravity of the brass, 200 cu, in, weigh

$$200 \times .0361 \times S$$
 lb.

Since the weight of the brass equals the sum of the weights of its parts, we have, $200 \times .0361 \times S = 47.6 \pm 13$ therefore

$$S = \frac{47.6 + 13}{200 \times .0361}$$
 or 8.39 the specific gravity

- 10. Forty pounds of copper and 27 lb, of zinc are alloyed to make brass. What is its specific gravity?
- 11. A piece of metal weighs 26.8 lb, in air and 23.4 lb, when entirely immersed in water. What is its specific gravity?
- 12. Find the weight of oil in a cylindrical tank 8 H in dram, and 16 ft. long if the specific gravity of the oil is .98.

- 13. A casting weighs in air 22.8 lb. When entirely immersed in oil whose specific gravity is .94 the easting weighs 19.6 lb. Find its specific gravity and its volume in cubic inches.
- 14. An alloy is made up of 30 cu. in. of tin and 48 cu. in. of copper. Find its specific gravity.

CHAPTER IV

SCREW THREADS

10. Thread Sections.—The shape and size of a thread are determined by the shape and size of its cross section taken in a plane passing through the axis of the screw. The dimensions of any particular kind of thread are fixed by stating the number of threads or sections in an inch, measured parallel to the axis of the screw, as, for example, 8 threads per inch, or simply 8 threads.

Fig. 17 shows the sections of four different screw threads. In the machine shop on regular work the U.S.



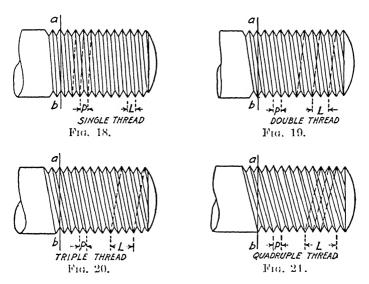
Fig. 17.

Std. Thread is used. In some practice bolts and studs 34 in. in diam, and above screwed into boilers have twelve sharp V threads per in., irrespective of the diameter of the bolt or stud.

11. Pitch and Lead.—Before looking into the method of constructing and calculating threads we should understand first the meaning of the "pitch" and "lead" of a screw. The "pitch" of a screw is the distance from the center of one thread to the center of the next. The "lead" of a screw is the distance the screw advances in one complete turn.

12. Single, Double, and Triple Threads.—A single-threaded screw, as shown in Fig. 18, is one which advances in one complete turn a distance (measured in the direction of the axis of the screw) equal to that from a point on one thread to a corresponding point on the next thread, or a distance equal to the "pitch."

A double-threaded screw, as shown in Fig. 19, ad-

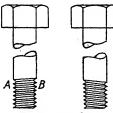


vances in one complete turn a distance equal to twice the "pitch." For a double thread the "lead" is therefore twice the "pitch." In a triple thread, as shown in Fig. 20, the advance for one complete turn equals three times the "pitch," or the "lead" equals three times the "pitch." Fig. 21 shows a quadruple thread.

13. Right- and Left-handed Threads. A right-handed thread is one in which the thread rises from left to right,

as from A to B in Fig. 22. Fig. 23 shows a left-handed thread.

Suppose now, for example, we were making a sketch of a threaded bolt, we should show the following. The total length of bolt including the head, the length of



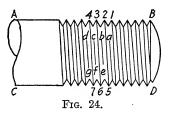
Figs. 22 and 23.

body including the threaded portion, length of threaded portion, diameter of bolt, the thread section, number of threads per in., whether thread is right- or left-handed and whether single, double or triple. If there are no standard tables available for finding the dimensions of the bolt

head, the thickness and short diameter of the head should be shown.

14. Construction of a Single, Right-handed V Thread, 8 Threads to an Inch.—Lay off on line AB, Fig. 24 (an element of the portion to be threaded), $\frac{1}{8}$ -in. distances as

1-2, 2-3, etc., each equal to the pitch of the screw. With the 60° triangle draw the sections 1a2, 2b3, etc. A line through point a parallel to line AB contains the points b, c, d, etc., at the bottoms of the threads. A perpendicular

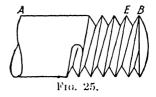


from point a to line CD gives point 5 on the *outside* of the thread. Similarly perpendiculars dropped from points b, c, d, etc., locate points on the outside of the thread on line CD. With the 60° triangle the sections 5e6, etc., are completed and the visible edges of the thread are drawn by connecting points b and e, 2 and 5, etc., as

shown. The edges of the thread form turns called "helices," a single turn being a "helix." These edges are, however, simply represented by straight lines which are used in most cases as in Fig. 24.

15. Construction of a Double, Left-handed V Thread, 4 Threads to an Inch.—In Fig. 25 portion AB is to be threaded. The "pitch" is here ¼ in., and the "lead" $^{1}2$ in. Since the thread is double, the outside points of the thread on opposite sides of the screw fall opposite each other. The construction is otherwise the same as for the single V thread.

In one complete turn the screw advances a distance equal to twice the "pitch," or a distance of ½ in, equal to the "lead" of the screw. The distance BE corresponds to the pitch. The



screw is left-handed, since the thread ascends from right to left looking toward the head of the bolt.

16. Construction of a United States Standard Thread. The U. S. Std. for screws and threads is that adopted by the U. S. Government and by the R. R. Master Mechanics and Master Car Builders Associations, also by leading tool builders and manufacturers. This thread has the same angle as the V thread or 60° thread. The dimensions of a thread for a pitch equal to 1 in, are shown in Fig. 26. The triangle ABC is equilateral. If each of its sides equals 1 in, the distance CD equals the square root of $[(1)^2 - ({}^1_2)^2]$ or the square root of $[4]^2 - ({}^1_2)^2]$ or the square root of $[4]^2 - (4]^2 - (4]^2 - (4]^2 - (4]^2 - (4]^2 - (4]^2 - (4)^2 -$

in., as shown in the figure, that is, $\frac{1}{8}$ of the pitch. To find the depth of a U.S. Std. thread of another pitch, divide these figures by the number of threads to the inch.

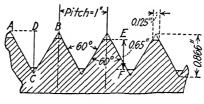


Fig. 26.

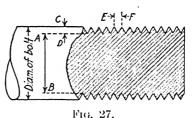
PROBLEMS

- 1. What is the lead of a single-threaded screw having 12 threads per in.? What is the pitch of this screw?
- 2. A double thread measures .125 in. between the points, what is its pitch and lead?
- 3. How many turns must a single thread of pitch .166 in. make to advance 11/4 in.?
- **4.** A double-threaded screw advances $1\frac{1}{2}$ in. in three complete turns. What is its pitch?
- 5. Find the depth of thread, and width of flats for a U. S. Std. Thread, with the following number of threads per in. 8, 12, 13, 18.
- 6. The number of threads per inch (U. S. Std.) corresponding to the diameter of the bolt are as follows:—Diam. of bolt ½ in.; threads per in., 13; diam. ½ in.; threads, 9; 1½ in. diam. threads, 7. For each of these cases calculate the diameter at root of thread and the area of bolt at root of thread, also the width of flat.
- 7. In a machine, three bolts each 1 in. in diam. with 8 U. S. Std. threads per in., are replaced by a single bolt. What must be the size of this bolt at root of thread to have the same area of metal at root of thread as the three 1-in. bolts?
- 8. What is the "pitch" and the "lead" of a triple thread which advances 2 in. in two complete turns?
- 9. Make a sketch of a double, right-hand V thread with a pitch of 1/8 in.

- 10. Find the size of a tap drill in nearest 64ths in, for a "V" thread 8 per in, on a bolt 1 in, in diam.
- 11. Make a sketch of a single right-hand U. S. Std. thread with S threads per in. What is the pitch and the lead in this case?
- 12. How does the number of threads per in, on a bolt and hence the depth of the thread affect the strength of the bolt? Would it be practicable to have only 4 threads per in, on a 1-in, bolt? Why with some companies are 12 sharp V threads per in, used in boiler work on any size bolt or stud above 34 in, in diam,?
- 13. What is the advantage of double and triple threads over a single thread? How does the number of threads per in, affect the "speed" of a bolt or screw? Which form of thread is used most in the machine-shop work, the V or U. S. Std.?

Referring to Fig. 27, showing the section of a V thread, the distance AB, or the diameter at the root of the thread

equals the diameter of the bolt minus twice the distance CD. The length CD is .86 of the distance EF, and the distance EF depends on the number of threads per in, of the screw. In an 8-thread



serew $EF \sim {}^{1}{}_{8}$ in, or .125 in., for a 10-thread screw EF =

 ${}^{1}_{10}$ in, or .10 in., for a T-thread.screw $EF = \frac{1}{T}$

From the above we have the rule for a V thread.

$$R \approx D \sim \frac{1.73}{T}$$

where R = Diam, at root of thread.

D Diam, of bolt (outside of threads).

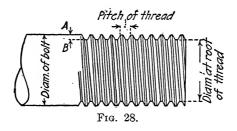
T No. of teeth or threads per in.

If in the case of bolts or study $^{3}4$ in, in diam, and above, screwed into boilers, 12 sharp V threads per in, are used,

of the diameter of the bolt or stud, the prebecomes in this case:

$$R = D - \frac{1.73}{12}$$
 or $R = D - 0.144$ in.
or $R = D - \frac{9}{64}$ in.

re of the U. S. Std. thread as in Fig. 28 the at the root of the thread equals the diameter t minus twice the distance AB. The distance \times the pitch. Therefore the diameter at the bethread



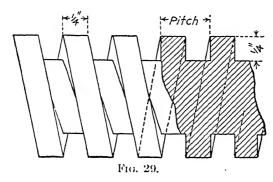
- = diameter of bolt $-2 \times 0.65 \times$ the pitch.
 - The pitch $=\frac{1}{T}$ where T is the number of threads per in.

Therefore diameter at root of thread for U. S. Std.

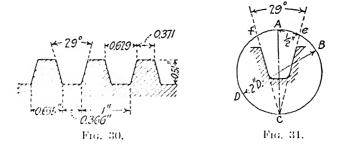
- = diameter of bolt $-\frac{1.30}{T}$
- 17. Construction of a Single, Square Thread, 2 Threads Per Inch. Fig. 29 Shows the Method of Construction.
- 18. Construction of an Acme Thread.—Square threads are not always easily cut, and there has been developed a standard, which is flat at top and bottom and has its sides inclined at an angle of $14\frac{1}{2}^{\circ}$ to the center line of

the thread section. Fig. 30 shows the section for such a thread and the dimensions of the threads for a pitch of 1 in.

The angle of 29° or the total angle formed by the sides



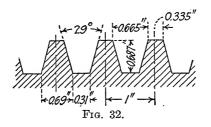
of an Acme thread as in Fig. 31 may be drawn with the use of a protractor or as shown in the figure which is drawn as follows:



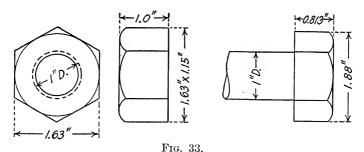
A circle ABCD is drawn 2 in, in diam, and also a vertical diam. AC. From A as a center and with a radius of $^{1}_{2}$ in., ares are drawn cutting the circle at points c and f. Connecting these points with point C by the straight

lines eC and fC gives the inclination for the sides of the threads. The angle eCf is 29° .

In a "Worm" thread as in Fig. 32 the angle is the same as for the Acme thread, that is, 29°, but the thread is about one-third (1/3) deeper than the Acme.



19. Miscellaneous Rules.—The width across flats of U.S. Std. hex. nuts equals $1\frac{1}{2} \times \text{Diameter of bolt} + \frac{1}{8}$ in. See Fig. 33. The width across corners of U.S. Std. hex. nuts equals $1.155 \times \text{width across flats}$. The



thickness of U. S. Std. hex. nuts (rough) is the same as diameter of bolt. Width across corners and flats of bolt heads is the same as for hex. nuts. The thickness of bolt heads equals half the width across flats of bolt head or $\frac{3}{4} \times \text{diameter}$ of bolt $+\frac{1}{16}$ in.

TABLE FOR U.S. STD. THREADS

Diam. of screw	Threads to inch	Pitch	Depth of thread	Diam. at root of thread	Width of flat
1/4	20	. 0500	.0325	.185	.0063
5/16	18	. 0556	.0361	.2403	. 0069
3/8	16	. 0625	. 0405	.2936	.0078
7/16	14	.0714	. 0461	.3447	.0089
1/2	13	.0769	.0499	.4001	.0096
⁹ 16	12	.0833	.0541	.4542	.0104
5/8	11	.0909	.0591	.5069	.0114
$\frac{3}{4}$	10	. 1000	. 0649	.6201	.0125
7∕8	9	.1111	.0721	.7307	. 0139
1	8	.1250	.0812	.8376	.0156
11/8	7	.1429	.0928	.9394	.0179
11/4	7	.1429	.0928	1.0644	.0179
13/8	6	. 1667	. 1082	1.1585	.0208
11/2	6	. 1667	.1082	1.2835	.0208
15/8	$5\frac{1}{2}$. 1818	.1181	1.3888	.0227
134	5	.2000	.1299	1.4902	.0250
17/8	5	. 2000	.1299	1.6152	.0250
2	41/2	.2222	. 1444	1.7113	.0278
$2\frac{1}{4}$	$4\frac{1}{2}$.2222	. 1444	1.9613	.0278
$2\frac{1}{2}$	4	.2500	. 1624	2.1752	.0313
234	4	.2500	. 1624	2.4252	.0313
3	312	.2857	. 1856	2.6288	.0357
31/4	$3\frac{1}{2}$.2857	. 1856	2.8788	.0357
$3\frac{1}{2}$	334	.3077	. 1998	3.1003	.0385
334	3	. 3333	.2165	3.3170	.0417
4	3	. 3333	.2165	3.5670	.0417
41/1	27/s	.3478	.2259	3.7982	.0435
41/2	234	. 3636	.2362	4.0276	.0455
434	25_8	.3810	. 2474	4.2551	.0476
5	$2\frac{1}{2}$. 4000	.2598	4.4804	. 0500
514	$2\frac{1}{2}$.4000	.2598	4.7304	.0500
$\frac{512}{2}$	$2\frac{3}{8}$.4210	.2735	4.9530	.0526
534	238	.4210	.2735	5.2030	.0526
6	$2\frac{1}{4}$. 4444	.2882	5.4226	. 0556

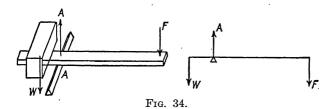
TABLD FOR SHARP V THREADS

Diam. of screw	No. threads per inch	Pitch	Depth of thread	Diam. at root of thread
1/4	20	.0500	.0433	. 1634
5∕16	18	. 0556	.0481	.2163
3/8	16	. 0625	.0541	.2667
½ 16	14	.0714	.0618	. 3140
$\frac{1}{2}$	12	.0833	.0722	. 3557
%16	12	. 0833	.0722	.4182
5⁄8	11	. 0909	.0787	. 4676
¹ / ₁₆	11	0909	.0787	. 5301
3/4	10	. 1000	.0866	. 5768
¹ 3⁄16	10	.1000	.0866	.6393
⅓ 8	9	.1111	.0962	. 6826
¹⁵ / ₁₆	9	.1111	.0962	.7451
1	8	.1250	.1083	.7835
11/8	7	.1429	. 1237	.8776
11/4	7	.1429	. 1237	1.0026
13/8	6	.1667	. 1443	1.0864
11/2	6	. 1667	.1443	1.2114
15/8	5	. 2000	.1733	1.2784
13/4	5	. 2000	.1733	1.4034
17/8	4½	.2222	.1924	1.4902
2	41/2	. 2222	.1924	1.6152
21/8	$4\frac{1}{2}$.2222	.1924	1.7402
$2\frac{1}{4}$	41/2	.2222	.1924	1.8652
23/8	$4\frac{1}{2}$. 2222	. 1924	1.9902
$2\frac{1}{2}$	4	. 2500	.2165	2.0670
25/8	4	.2500	.2165	2.1920
23/4	4	.2500	.2165	2.3170
21/8	4	.2500	.2165	2.4420
33	$3\frac{1}{2}$. 2857	.2474	2.5052
1/8	$3\frac{1}{2}$.2857	.2474	2.6301
31/4	$3\frac{1}{2}$. 2857	.2474	2.7551
33/8	31/4	. 3077	.2666	2.8418
3½	$3\frac{1}{4}$.3077	.2666	2.9668
35/8	31/4	.3077	.2666	3.0918
3¾	3	. 3333	.2886	3.1727
37/8	3	. 3333	.2886	3.2977
4	3	. 3333	.2886	3.4227

CHAPTER V

CALCULATION OF LEVERS

20. Definitions and Problems.—A lever is a rod, bar or beam of any shape, either straight or curved, which is free to turn about a fixed point called a "fulcrum." The fulcrum may be any form of support, prop or bearing.



Levers are divided into three classes according to the positions of the loads, pressures, or forces acting on the lever with respect to the position of the fulcrum. In

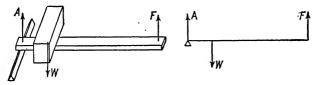


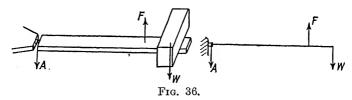
Fig. 35.

the first class, Fig. 34 the fulcrum A is between the weight (W) and the force (F) applied to lift the weight.

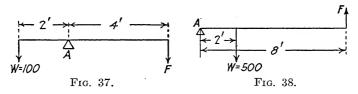
In the second-class lever Fig. 35 the weight W is between the fulcrum A and the force F.

In the third-class lever the force F is between the fulcrum A and the weight W, as in Fig. 36.

In working out lever problems we find what are called the "moments" of the forces or pressures or weights acting on the lever. In Fig. 37 which shows a lever of the first class the force F, 4 ft. from the fulcrum A,



raises the weight W of 100 lb. located 2 ft. from A. Multiplying F by 4 lb., its distance from A gives the moment of F about the "fulcrum" A. Similarly $W \times 2$ ft. is the moment of W about the "fulcrum" A. To find what force is necessary at F to raise a weight of 100 lb. at W, we multiply $F \times 4$ ft. giving the moment of



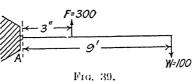
F about A and place this product equal to 100×2 which is the moment of the weight of 100 lb. about A. Then

therefore
$$100 \times 2 = F \times 4$$

$$200 = 4F$$
 and
$$F = \frac{200}{4} \text{ or } 50 \text{ lb.}$$

Therefore 50 lb. pull at F will lift 100 lb. at W neglecting friction and the weight of the lever itself. The rule in any case is, find the moment of the force about the

fulcrum and place this equal to the moment of the weight about the fulcrum and solve for the quantity desired.



In Fig. 38 to find a

force F necessary to lift a weight of 500 lb. at the distance given we have $500 \times 2 = F \times 8$ therefore

$$1000 = 8F$$

 $F = 125 \text{ lb.}$

and

In Fig. 39 which represents a lever of the third class a force of 300 lb. at F will lift only 100 lb. at W, since taking the "moments" about the fulcrum A we have $300 \times 3 = W \times 9$.

Therefore and

$$900 = 9W$$

 $W = 100 \text{ lb.}$

PROBLEMS

What class of lever does Fig. 40 show and why? From the rules already given and using Fig. 40, fill out the following table,



Fra. 40.

giving also sample calculations to show how you obtained your results. Neglect the weight of the lever itself.

No.	W, lb.	• F, lb.	а	ь
$_{2}^{1}$	30	50	1 ft. 2 ft.	3 ft. 8 ft.
3	60	30		6 ft.
4 5	80 18	10	2 ft. 3½ ft.	4¾ ft.
6		40	8 in.	2 ft.

What class of lever does Fig. 41 show and why? Using Fig. 41 fill out the following table, giving also sample calculations to

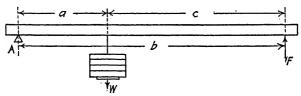


Fig. 41.

show how you obtained your results. Neglect the weight of the lever itself.

No.	Flb.	W lb.	а	ь	c
7	50	200	8 ft.		
8	30		6 ft.		2 ft.
9		10	1 ft.	5 ft.	
10	30	60		4 ft.	
11		20	6½ ft.		2 ft.
12	27			39 in.	27 in.

What class of lever does Fig. 42 show, and why? Using Fig. 42 fill out the following table, giving also sample calculations to show how you obtained your results. Neglect the weight of the lever itself.

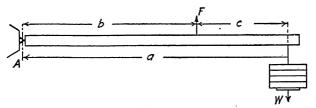
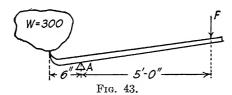


Fig. 42.

No.	W, lb.	F, lb.	a	ь	c
13	50	200	8 ft.		
14	30		6 ft.		2 ft.
15		10	10 ft.	5 ft.	
16	30	60		4 ft.	
17		20	6½ ft.		2 ft.
18	27			39 in.	27 in.

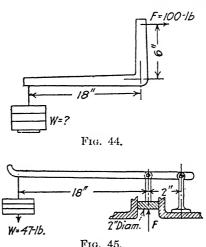
19. As shown in Fig. 43, what downward force at F is necessary to start the stone if the resistance offered is 300 lb. If the stone



could not be moved with the bar as shown how could a greater force or pressure be produced on the stone, using the same bar? What class of lever does this represent?

- 20. In Fig. 44 the lever arms are bent at right angles to each other, allowing the effect of the weight W to be produced in a horizontal direction at F. With the lever arms as shown, what weight is necessary at W to produce a pull of 100 lb. at F?
 - 21. In Fig. 45, showing a safety valve for a stationary boiler,

at what pressure will the boiler "blow off" with the size of valve and lever arms as shown? What class of lever does this show?



21. Calculation for Levers Taking into Account the Weight of the Lever Itself in Each Case.—If the lever shown in Fig. 46 weighs .5 lb. per in. of length the portion to the left of the fulcrum weighs $8 \times .5$ or 4 lb.

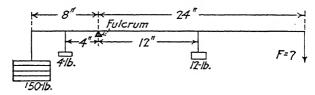
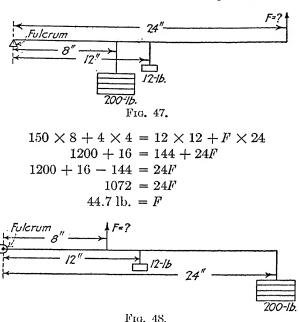


Fig. 46.

The portion to the right of the fulcrum weighs $24 \times .5$ or 12 lb. These weights should be taken at the *center of gravity* of the respective parts of the lever.

For example, the entire weight of the 8-in. length is

considered as concentrated at a point 4 in. from the fulcrum, that is, at the center of gravity of the 8-in. length. Likewise the 12 lb. is considered as concentrated at a distance of 12 in. from and at the right of the fulcrum. We now have four forces to deal with and our equation becomes:



In Fig. 47, the lever weighs .5 lb. per in. of length or 12 lb. total weight. This acts at a distance of 12 in. from the fulcrum, that is, at the *center of gravity* of the whole lever. The equation then becomes

$$200 \times 8 + 12 \times 12 = F \times 24$$

 $1744 = 24F$
 $72.6 \text{ lb.} = F$

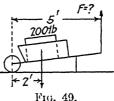
In Fig. 48, the lever weighs 12 lb. and the equation becomes

$$12 \times 12 + 200 \times 24 = 8F$$

 $4944 = 8F$
 $618 \text{ lb.} = F$

PROBLEMS

- 22. In a lever of the first class a weight of 250 lb. 6 in. from the fulcrum can be balanced by what force applied 112 ft. from the fulcrum?
- 23. Work out the above problem if the lever itself weighs 4 lb. per ft. of length.



- 24. Find the force F required to lift the weight of 200 lb. on the wheelbarrow as shown in Fig. 49.
- 25. From Fig. 50 find the force which can be exerted at F through the air cylinder and system of levers shown.

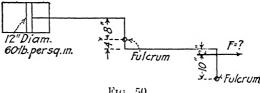
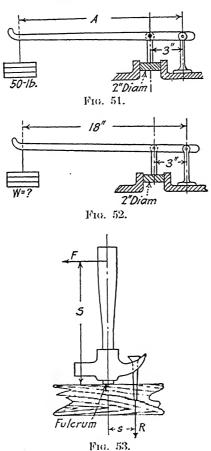


Fig. 50.

26. From Fig. 51, showing a safety valve for a stationary boiler, what must be the length of the arm "A" in order that the boiler will "blow off" at 150 lb. pressure per sq. in.? What class of lever does this sketch show?

27. From Fig. 52, showing a safety valve for a stationary boiler, what weight must be placed at W in order that the boiler will "blow off" at 150 lb. pressure per sq. in.?



28. In using an ordinary claw hammer for pulling a nail as shown in Fig. 53, we are using a first-class lever. Suppose as shown

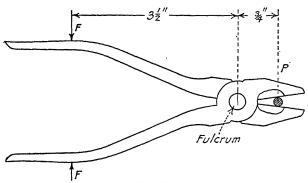


Fig. 54.

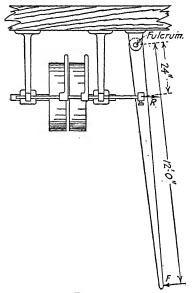
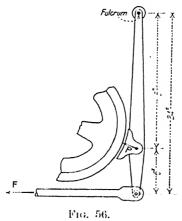


Fig 55.

a force of 40 lb. is exerted at "F," we find the resistance of the nail at the start by calculating a first-class lever, whose arms are bent up at right angles to each other. If the distance s=2 in. and S=9 in., calculate the resistance "R" of the nail at the start.

- 29. The pliers and wire cutters shown in Fig. 54 form a first-class lever. If the total force on the handle of the pliers is 10 lb. at a distance of $3\frac{1}{2}$ in from the fulcrum, how much pressure is produced on the wire at "P" in cutting it? The wire is $\frac{3}{4}$ in from the fulcrum.
- **30.** Fig. 55 shows a "belt shifter." If the resistance at "R," required to be overcome in shifting the belt is 35 lb., what force is required at "F" to move the belt with the lever arms shown?



31. If in shifting the belt it is necessary to move the shifter rod **4** in, how far does the end of the long arm move, that is, the end on which the force "F" is exerted?

In Fig. 56 is shown a second-class brake lever with brake shoe and brake rod. The motion of the brake shoe is to the motion of the pull rod as 18:24 with the lengths of lever arms as shown. For example if the

brake rod moves 4 in. the shoe movement is found as follows:

$$\frac{\text{Movement of brake shoe}}{\text{Movement of pull rod}} = \frac{18}{24} \text{ or } \frac{\text{movement of brake shoe}}{4}$$
$$= \frac{18}{24}$$

That is, movement of brake shoe = $4 \times \frac{18}{24} = 3$ in.

Also if the braking force through the brake shoe is called "P" we have the proportion $\frac{P}{F} = \frac{24}{18}$ from which

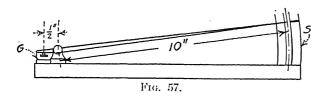
$$P = \frac{F \times 24}{18}.$$

Example.—If the pull on the rod is 4800 lb., that is, if F = 4800 lb. then $P = \frac{4800 \times 24}{18}$ or 6400 lb.

- 32. In the brake lever shown in Fig. 56, if the distance from the fulcrum to brake shoe is 24 in. and the whole effective length of lever is 30 in. what is the movement of brake shoe compared with that of the brake rod? What pull is necessary to produce a pressure of 8000 lb. against the wheel?
- **33.** Find the length of lever arms for a brake lever as shown in Fig. 56 such that a pull of 800 lb. on the brake rod will produce a pressure of 4000 lb. of the brake shoe against the wheel. In this case what is the ratio of movement of brake shoe and brake rod?

Levers used to Increase or Decrease Motion.—Levers are often used to increase motion which of itself is too small to read easily as in the case of an indicator as shown in Fig. 57. With this instrument the piece to be measured is put in at "G." The instrument is "calibrated" or compared with a standard gauge to find the reading on the scale "S" for a given gauge. When the piece is put in at "G" any movement of the pointer above or below the gauge point on "S" shows how much too large or too

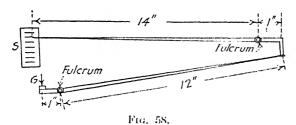
small the piece is that is being measured. The reading of the scale for different pieces tested will depend on the ratio of the arms of the lever used. If the short arm is .5 in. and the long arm 10 in. as shown the ratio of the motion of the ends of the lever is as 1:20. If for



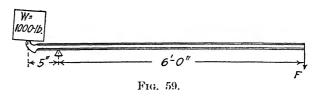
example in testing a piece the end of the long lever moves .2 in. beyond the standard gauge mark, the work being measured is one-twentieth of .2 or .01 in. from the standard.

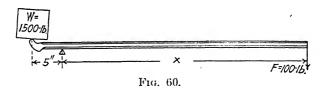
PROBLEMS

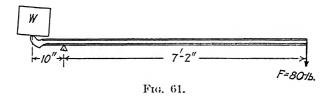
34. In the indicator shown in Fig. 57 the pointer indicates on scale "S" a distance of .48 in, beyond the gauge mark. How much is the piece being measured out of gauge?

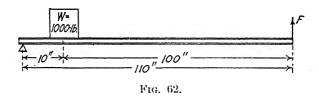


35. An indicator similar to that shown in Fig. 57 has lever arms of .75 in, and 18 in. If a piece being measured is .025 in, from standard gauge, how far will the pointer move on scale "S" from the standard gauge mark?









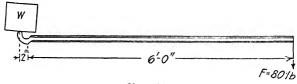
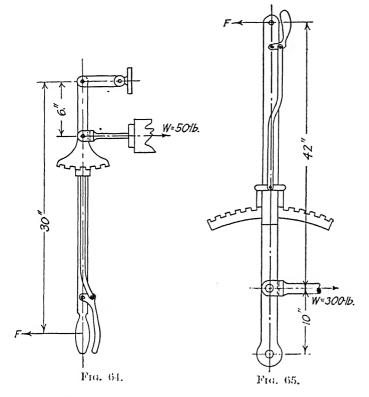


Fig. 63.

- **36.** In the lever indicator system shown in Fig. 58, find the motion produced at "S" by a motion of .040 in. at G.
- 37. Show how you could redesign the levers so that a motion of .040 in. at G would produce at "S" twice the motion calculated in the problem above.



- 38. In Fig. 59, find the force F necessary to raise the weight W.
- **39.** In Fig. 60, find the length of the lever arm X so that the force of 100 lb, will balance the load W of 1500 lb,
- **40.** In Fig. 61, find the weight W that can be raised by the force of 80 lb.

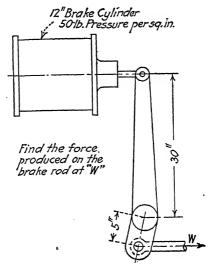


Fig. 66.

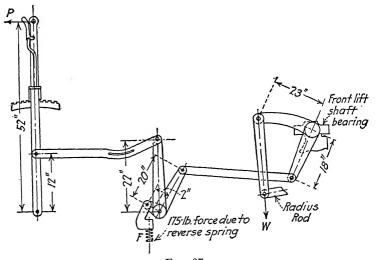


Fig. 67.

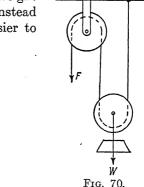
- **41.** In Fig. 62, find the force F required to raise the weight W.
- 42. In Fig. 63, showing a pinch bar 6 ft. long with 2-in. nose, find the weight W that can be raised.
- 43. In Fig. 64, showing a throttle lever, find the force F required to move the throttle.
- 44. Fig. 65 shows a reverse lever. Find the force F required to reverse the engine.
- 45. Fig. 66 shows a brake cylinder. Find the force produced on the brake rod at W by the pressure of 50 lb. per sq. in. in the cylinder.
- 46. With a load at W, Fig. 67, of 400 lb. due to lifting the link blocks, radius rod ends and moving the valves, what will be the force required at P to move the reverse lever backward taking into account the action of the reverse spring, but neglecting the friction of the reversing gear?

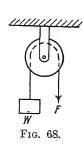
CHAPTER VI

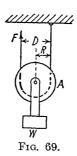
PULLEYS (BLOCK AND TACKLE)

22. Arrangement of Pulley Systems.—Pulleys or blocks (Fig. 68) are used to change the direction of application of a force, as well as to reduce the value of the force necessary to lift a given weight or overcome a given resistance. Pulleys are in fact rotating levers.

With a single pulley as in Fig. 68, the force F applied to raise the weight W, can be exerted downward instead of upward. This makes it easier to







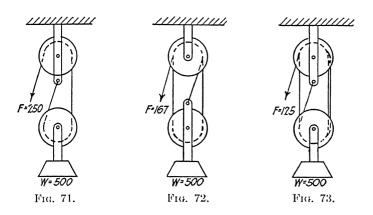
raise the weight. In this case the force required at F is equal to the weight lifted at W, neglecting friction.

This arrangement forms a *lever* of the first class. Compare this with a first-class lever.

In the case of a single movable pulley, as in Fig. 69, the force necessary at F is one-half the weight lifted at

W, neglecting the effect of friction. This arrangement forms a lever of the second class and we have the weight $W \times \text{radius}$ of pulley $= F \times \text{diameter}$ of pulley. Therefore to lift 100 lb. at W requires a force of 50 lb. at F. Compare this figure with a second-class lever.

Fig. 70 shows a fixed and a movable pulley, or a combination of the arrangements shown in Figs. 68 and 69. The force required at F to lift a weight W is



equal to one-half of the weight neglecting friction of the rope and pulleys. When the force F is exerted through a distance of 2 ft., the weight is lifted 1 ft. A force of 50 lb. at F will raise a weight of 100 lb. at W. The friction of the rope on the pulleys and the pulleys on their bearings is usually small compared to the weights ordinarily raised, and this friction is reduced by the use of grease or oil on the bearings.

In the following arrangements of blocks, Figs. 71, 72, and 73, the force required to raise a given weight

or vice versa is found as follows: Rule.—The force in pounds multiplied by the distance in feet through which it moves, equals the weight in pounds lifted, multiplied by the distance through which it moves, and another rule which also applies for any number of pulleys in either block is: Divide the weight to be lifted by the number of single strands or lines of rope extending down to the movable pulley, to find the force F. The distance through which the weight moves equals the distance through which the force F moves divided by the number of single lines of rope extending down to the movable block. In Fig. 71 a force of 250 lb. is necessary to lift a weight of 500 lb., neglecting friction. For each of the arrangements shown the force necessary to raise a weight of 500 lb. is given.

PROBLEMS

- 1. With a single movable pulley as in Fig. 69, what force is necessary at F to raise a weight of 300 lb.? What advantage has the arrangement shown in Fig. 70 over this one using only the single movable pulley?
- 2. In Fig. 71 what force at F is necessary to lift 100 lb. at W? What weight at W will just balance a downward pull at F of 230 lb.?
- 3. With arrangement as shown in Figs. 72 and 73, does the size of pulleys have anything to do with the force required to lift a given weight?
- **4.** In Fig. 72, how much force is necessary at F to raise a weight of 800 lb. at W?
- **5.** In Fig. 72, what weight at W will just balance a pull at F of 200 lb. if the lower block and hook together weigh 20 lb.?
- 6. In Fig. 73, what weight could a man weighing 150 lb. raise by hanging on the rope at the end F?

If the weight was raised 1 ft., through how many feet does the man move?

7. What force at F will a weight of 1000 lb. at W just balance

with the arrangement of Fig. 73, if the lower block and hook together weigh 26 lb.?

The Differential Pulley.—For lifting heavy weights by hand a differential pulley is often used.

As shown in Fig. 74, this consists of two pulleys A and B rigidly fastened together and rotating as one about a fixed axis C. An endless chain passes over both pulleys, and one of the loops of the chain passes under and supports the lower or movable block D. The other loop of the chain hangs freely and is the loop upon which

one pulls in raising a weight on the hook E. The rims of the pulleys are grooved and carry lugs to prevent the chain from slipping. If the radii of the large and small fixed pulleys are R and r respectively, "P" the pulling force exerted on the chain and "W" the weight lifted, the rule for the pulley is as follows, leaving out friction:

or
$$P \times R = W \times \frac{1}{2}(R - r)$$

$$\frac{W}{P} = \frac{R}{\frac{1}{2}(R - r)}$$

As used a differential pulley can actually lift only about 25 to 35 per cent. of the weight calculated for a given pull

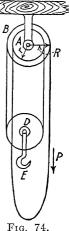


FIG. 74

from the formula stated above. This means therefore that to lift a given weight requires actually from three to four times the pull calculated by the formula which does not take account of friction.

Example.—What force is necessary to lift a weight of 800 lb. with a differential hoist having pulleys 10 in. and 8½ in. in diam. and an efficiency of 25 per cent.?

Solution.—
$$P \times R = W \times \frac{1}{2}(R - r)$$

that is $P = \frac{W \times \frac{1}{2}(R - r)}{R}$
or $P = \frac{800 \times \frac{1}{2}(5 - 4\frac{1}{4})}{5}$
 $= \frac{800 \times \frac{1}{2} \times \frac{3}{4}}{5}$
or $\frac{800 \times \frac{3}{8}}{5}$ or 60 lb.

Since the efficiency is only 25 per cent. the actual pull required is $\frac{60}{.25}$ or 240 lb.

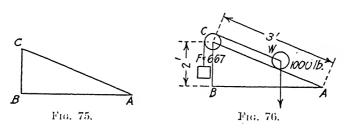
PROBLEMS

- 8. From the rule for a differential pulley find the force necessary to lift a tank weighing 280 lb. The diameters of the pulleys being 12 in. and 10 in. and the efficiency 30 per cent.
- 9. In a differential hoist with pulleys 11 in. and $9\frac{1}{2}$ in. and an efficiency of 53 per cent., find what weight a 150-lb. man can raise by putting his own weight on the chain.

CHAPTER VII

THE INCLINED PLANE AND WEDGE. THE SCREW JACK

23. The Inclined Plane and Wedge.—A block whose sloping surface is less than 90° may be considered an "inclined plane." In Fig. 75 which shows such an inclined plane, AB is the "base," BC the "height" and AC the "face" or "sloping surface." With the use of the inclined plane a given resistance can be overcome with a smaller force than if the plane were not used. For ex-



ample, in Fig. 76, suppose we wished to raise a weight of 1000 lb. through the vertical distance BC = 2 ft. If this weight were raised vertically and without the use of the plane the force of 1000 lb. would have to be exerted through the distance BC. If, however, the inclined plane is used and the weight is moved over its face AC, a force of only $\frac{2}{3}$ of 1000 lb. or 667 lb. is necessary, although this force is exerted through a distance AC which is greater than distance BC. With the

inclined plane, we, therefore, require a smaller force which must be exerted through a greater distance to do a certain amount of work.

Letting F represent the force required to raise a given weight on the inclined plane, and W the weight to be raised, we have the proportion:

$$\frac{F}{W} = \frac{\text{height of plane}}{\text{length of sloping surface of plane}}$$

or for Fig. 76

$$\frac{F}{W} = \frac{2}{3} \text{ and } \frac{F}{1000} = \frac{2}{3}$$

therefore $F = \frac{2}{3} \times 1000$ or 667 lb.

Stated in words the rule for the "Inclined Plane" is:

The force required to raise a given weight over the plane is to the weight to be raised as the height of the plane is to the length of the sloping surface.

To find the force required, multiply the weight on the plane by the height of the plane and divide by the length of the face of the plane.

To find the weight which a given force will raise, multiply the force by the length of the face of the plane and divide by the height of the plane.

These rules do not include the effect of friction on the plane.

As shown in Fig. 77, a "wedge" consists of two inclined planes with their bases together. The force F used to drive the wedge acts parallel to the center line of the wedge.

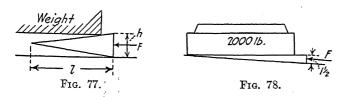
The principle of the wedge is the same as that for the inclined plane. In this case the wedge in being driven in is the same as if the weight being raised moved along a stationary inclined plane. Leaving out the effect of

friction on the sides of the wedge, the work done in driving the wedge equals the force exerted, F times l or Fl. The work done in raising the weight equals the weight lifted "W" times the distance lifted "h" or Wh. Then leaving out friction

$$Fl = Wh \text{ or } \frac{F}{W} = \frac{h}{l} \text{ or } F = W \times \frac{h}{l}$$

Therefore the force required is only $\frac{h}{l}$ times the weight lifted.

Example.—In Fig. 78, a wedge 12 in. long has a taper



of $1\frac{1}{2}$ in. per ft. and is used to raise a machine base weighing 1 ton. The force F required is found as follows:

$$F = \frac{W \times h}{l} = \frac{2000 \times 1.5}{12} = 250 \text{ lb.}$$

The "mechanical advantage" in using the wedge is therefore $\frac{2000}{250}$ or 8. That is, the force required is only $\frac{1}{8}$ of the weight lifted.

PROBLEMS

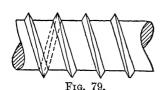
1. A weight of $2\frac{1}{2}$ tons is to be raised by a wedge having a taper of 1 in. per ft. What force is required to drive the wedge and what is the mechanical advantage leaving out the effect of friction?

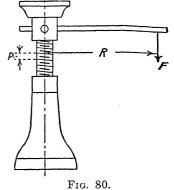
- 2. To wedge up a weight with a wedge tapering 2 in. per ft. requires a force of 500 lb. Neglecting friction, find the weight raised.
- 24. The Screw Jack.—One of the most common applications of the principle of the inclined plane is in the ordinary screw thread, which is nothing but an inclined plane wound around a cylinder.

If we cut out a wedge-shaped piece from thick paper and wind it around a cylinder we will see that the sloping

side forms a thread. The form of this thread as in Fig. 79 is called a "helix." All nut, bolt, jack, and screw threads are circular or spiral wedges.

Fig. 80 represents a screw





jack which is used to overcome a heavy pressure or raise a heavy weight at W by a much smaller force F applied at the handle. R represents the length of the handle and P the pitch of the screw, or the distance the screw advances in one complete turn.

Neglecting friction the following rule is used: The force F multiplied by the distance through which it moves in one complete turn is equal to the weight lifted times the distance through which it is lifted in the same time. In one complete turn the end of the handle describes a

circle of circumference $2\pi R$. This is the distance through which the force F is exerted.

Therefore from the rule above

$$F \times 2\pi R = W \times P$$

 $\pi = 3.14$ and $F = \frac{W \times P}{2\pi R}$

Suppose in Fig. 80, R = 18 in., $P = \frac{1}{8}$ in. and the weight to be lifted is 50 tons or 100,000 lb., the force in lb. required at F is then found as follows:

$$F = \frac{W \times P}{2\pi R}$$

$$F = \frac{100,000 \times \frac{1}{8}}{2\pi 18} \text{ or }$$

$$F = \frac{100,000}{8} \times \frac{1}{2\pi 18}$$

Cancelling we get $F = \frac{3125}{\pi 9} = \frac{3125}{28.3} = 110$ lb. necessary.

This means then that neglecting friction 110 lb. at F will raise 100,000 lb. at W, but the weight lifted moves much slower than the force applied at F.

PROBLEMS

- 3. What weight can be held by a force of 100 lb. on a plane which rises 4 ft. in every 5 ft. measured horizontally?
- 4. The height of an inclined plane is 10 ft. and its length is 20 ft. What weight will a force of 25 lb. at F in Fig. 76 hold up on the plane?
- 5. What counterbalance would be necessary to hold an empty coal car weighing 500 lb. on an incline 20 ft. long and 8 ft. high?
- 6. A screw jack has a single thread 8 threads per in, and the handle is 2 ft. long. What force is necessary at F to raise a weight of 2500 lb., neglecting friction?

7. What effect has increasing the pitch P, on the force necessary to raise a given weight if the length R does not change?

8. It is desired to jack up a machine with a square base and weighing 5 tons by four jacks, one at each corner. If the jacks have 6 threads per in. and $1\frac{1}{2}$ -ft. handles, and the weight is evenly distributed, what force is necessary at F?

9. What effect has increasing the length of handle on the force F necessary to raise a given weight if the same pitch is used?

10. A force of 50 lb. at F raises a weight of 3768 lb. at W with $R = 1\frac{1}{2}$ ft. What is the pitch of the screw?

CHAPTER VIII

GEARS, LATHE GEARING

25. Definitions.—A gear, or toothed wheel, when in operation, may be considered as a lever of the first class with the addition that it can be rotated continuously instead of rocking back and forth through a short distance. What we should learn here is the relation between the number of teeth, the diameter, and the speed of gears. Fig. 81 shows the ends of two shafts

A and B connected by 2 gears of 24 and 48 teeth respectively. Notice that the larger gear will make only one-half turn while the smaller makes a complete turn. That is, the ratio of speeds of the larger to the smaller is as 1 is to 2, or expressed as a proportion.

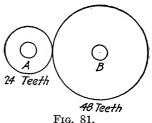


Fig. 81.

Speed of B: Speed of A = 1:2 and the ratio between the speeds and the number of teeth written as a proportion is

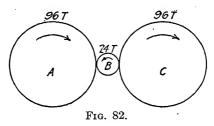
Speed of B: Speed of A = Teeth of A: Teeth of B. If in Fig. 81 gear A had 24 teeth and B 96 teeth, the smaller gear would make four turns while the larger makes 1, or,

Speed of A: Speed of B = 96:24, that is, the gear with the smaller number of teeth must turn the faster.

Of two gears running together that one is called the

driver which is nearer the source of power and the second gear which receives power from the driver is called the follower.

26. Gear Trains (Simple Gearing).—In the case of gear trains there may be several drivers and several followers. When the teeth on a gear turn in the same direction as the hands of a clock the motion is called right handed or "clockwise," and when in the opposite direction, left handed, or "counter-clockwise." In Fig. 82 suppose the three gears A, B, and C have 96, 24, and 96 teeth respectively.



When gear A turns once right handed, gear B turns 4 times left handed and gear C turns once right handed. Hence gear B does not change the speed of C from what it would have been if geared directly to A, but it changes its direction from left handed to right handed.

The ratio of speeds of the first and last gears in a train of simple gears is not changed by putting any number of gears between them.

27. Gear Trains (Compound Gearing).—Fig. 83 shows "compound gears" in which there are two gears on the middle shaft. Gears B and D rotate at the same speed since they are keyed to the same shaft. If the gears have a number of teeth as shown by the numbers in the

figure and gear A makes 100 r.p.m. right handed, gear B turns 400 r.p.m. left handed, also gear D turns 400 r.p.m. left handed and gear C turns 1200 r.p.m. right handed.

28. Lathe Gears for Screw Cutting (Simple Gearing).

—Modern lathes are equipped with a plate which gives the gears required for cutting different screw threads. To find the gears required in any case however it is merely necessary to understand the principle of a "simple" or a

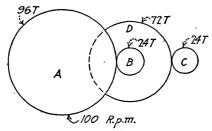


Fig. 83.

"compound" gear train depending on whether the lathe is simple or compound geared as explained in the following:

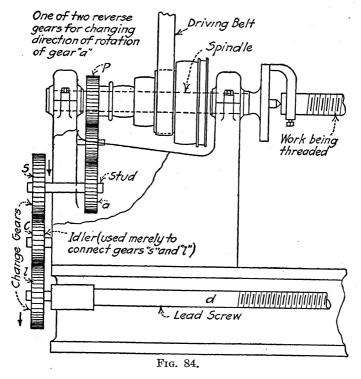
Fig. 84 shows a "simple geared" lathe with the names of important parts indicated. The lathe carriage carrying the cutting tool is moved by the lead screw which has usually 2, 4, 6, or 8 threads per in. If the lead screw has 4 threads per in., each complete revolution of this screw moves the carriage and hence the cutting tool \(\frac{1}{4} \) in. along the work being threaded, and 4 turns moves the tool 1 in. along the work. If now we wish to cut 12 threads per in., the spindle must revolve 12 times while the lead screw revolves 4 times. That is,

$$\frac{\text{turns of spindle}}{\text{turns of lead screw}} = \frac{12}{4} \text{ or as } \frac{3}{1}$$

The gears on the lead screw and on the spindle must therefore have a ratio of teeth of 3 to 1, or

$$\frac{\text{teeth on lead screw gear}}{\text{teeth on spindle gear}} = \frac{3}{1}$$

since the lead screw turns the slower.



If the gear a on the stud has the same number of teeth as gear p on the spindle, we have the rule

$$\frac{\text{teeth of lead screw gear}}{\text{teeth of stud gear}} = \frac{3}{1}$$

The idler gear (c) is used to connect gears S and l, and its number of teeth does not influence the speed ratio of the spindle and lead screw. The same is true of the gear connecting the spindle with the stud.

The following is the rule: The number of threads to be cut per in. equals the product of the number of threads per in. in the lead screw with the number of teeth in each driven gear and divided by the product of the number of teeth in each driving gear. In the figure shown, if the lead screw has 4 threads per in., gear p on the spindle 24 teeth, gear a on the stud 48 teeth, gear a 36 teeth, and gear a 172 teeth, the threads cut per in. equal $\frac{4 \times 48 \times 72}{24 \times 36}$ or 16.

Using the letters on the gears to indicate their number of teeth we have the rule, if n = the number of threads cut

per in. and d =the number of threads per in. of lead screw $n = \frac{d \times a \times l}{p \times s}$

For another example:

if d= threads per in. of lead screw =2 a= teeth in gear on stud =48 p= teeth in gear on spindle =24 s= teeth in outside gear on stud =24 l= teeth in lead screw gear =72Then $n=\frac{2\times48\times72}{24\times24}=12$ threads per in.

Suppose on the other hand we wished to find gears "s" and "l" to cut 8 threads per in. with a lead screw

having 6 threads per in. we have from the rule,

$$n = \frac{d \times a \times l}{p \times s}, \text{ and } \frac{l}{s} = \frac{n \times p}{d \times a}, \text{ from which}$$
$$\frac{l}{s} = \frac{8 \times p}{6 \times a} = \frac{8}{6} \times \frac{p}{a}$$

to find the ratio $\frac{l}{s}$ we must choose gears p and a. If we take p as 36 teeth and a 24 teeth, we have

$$\frac{l}{s} = \frac{8}{6} \times \frac{36}{24} = \frac{2}{1}$$

that is gears l and s must have a ratio of 2:1, hence we can put a 48-tooth gear on the lead screw and a 24-tooth gear as the outside gear on the stud and 8 threads per in. will be cut.

29. Lathe Gears for Screw Cutting (Compound Gearing).—In the compound geared lathe, as shown in Fig. 85, there are two change gears of different sizes on the spindle between the gear "s" on the stud and the gear "l" on the lead screw. The rule to be used in this case is as follows:

The threads to be cut per in. equal the product of the threads per in. of the lead screw and the teeth in each of the driven gears, divided by the product of the number of teeth in each of the driving gears. Representing the number of teeth of the gears by the letters on the gears we have, if "n" is the threads cut per in. and "d" the number of threads per in. of the lead screw.

$$n = \frac{d \times a \times l \times e}{p \times s \times f}$$

For example, suppose the lead screw has 5 threads per in. and the gears have a number of teeth as follows: $a=28,\ p=28,\ l=72,\ f=36,\ s=24,\ e=24,$ the number of threads cut per in. according to the rule becomes

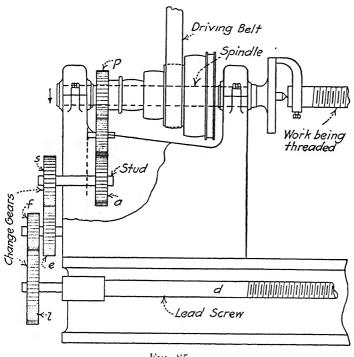


Fig. 85.

$$n = \frac{d \times a \times l \times e}{p \times s \times f} = \frac{5 \times 28 \times 72 \times 24}{28 \times 24 \times 36}, \text{ or } 10.$$

Working the other way round, if we wish to find the gears to use to cut a given thread we proceed as follows:

If d = 5 and the threads cut per in. are to be 14, we have

$$14 = \frac{5 \times a \times l \times e}{p \times s \times f}, \text{ from this}$$

$$\frac{14}{5} = \frac{a \times l \times e}{p \times s \times f}$$

The ratio of the gears must therefore be as 14 is to 5. Suppose by trial we take e = 28 and s = 30 teeth, we then have

$$\frac{14}{5} = \frac{a \times l \times 28}{p \times 30 \times f} \text{ or } \frac{14 \times 30}{5 \times 28} = \frac{a \times l}{p \times f}$$

that is $\frac{a \times l}{p \times f} = \frac{3}{1}$, the remaining ratio.

To provide this ratio we may take f = 24, l = 72, and a and p even gears as, for example, 36 teeth each. We then have

$$\frac{a \times l}{p \times f} = \frac{36 \times 72}{36 \times 24} \text{ or } \frac{3}{1}$$

PROBLEMS

1. If 8 threads are to be cut per in. and the lead screw has 6 threads per in., what must be the ratio of revolutions for the spindle and lead screw?

2. In a simple geared lathe calculate the size of change gears to use to cut 14 threads per in. if the lead screw has 5 threads per in.

3. In a lather the ratio of revolutions of lead screw and spindle is 1/3. If the pitch of the thread on the lead screw is 1/4 in., how many threads are cut for 16 revolutions of the lead screw?

4. In a compound geared lathe calculate the size of change gears to use to cut 18 threads per in. if the lead screw has 4 threads per in.

In working out the following problems think first of the relative sizes of the gears, which is the driver and which the follower, and remember that of two gears the larger must turn slower than the smaller. The diameters of gears are proportional directly to their number of teeth and inversely proportional to their speeds.

That is, if two gears A and B have pitch circles of diameters 12 and 24 in., respectively, and gear A has 48 teeth, gear B must have 96 teeth and if the speed of A is 100 revolutions per minute (r.p.m.) the speed of B is 50 r.p.m.

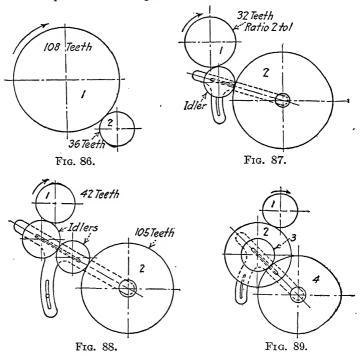
PROBLEMS

- **5.** If two gears A and B have 24 and 72 teeth respectively, and gear A turns right handed, in which direction will gear B turn? While gear A turns three times, how many times will B turn?
- 6. If two shafts are to be connected by gears of 48 and 36 teeth, and the faster makes 900 r.p.m. what speed does the other make?
- 7. Two gears have one 120 and the other 90 teeth. What is the ratio of the diameters of their pitch circles and of their speeds?
- 8. Two gears have pitch circles 76 in. and 19 in. in diam. What is the ratio of their speeds?
- 9. It is desired to run a shaft at 200 r.p.m. from another running at 500 r.p.m., what gear must be used on the second shaft if the gear on the first shaft has 150 teeth?
- 10. If 7 gears work in a train in what direction will the last one turn if the first turns left handed? How could you change the direction of rotation of the last gear?
- 11. A train of gears is made up of 4 gears with the number of teeth as follows: 72, 48, 32, and 24. If the first gear makes 20 r.p.m. right handed, in what direction and at what speed will the last gear move?
- 12. A train of gears is made up of 5 gears with the number of teeth as follows: 48, 36, 24, 60, and 48. If the first gear makes 30 r.p.m. left handed, in what direction and at what speed will the last gear move?
- 13. A main shaft runs at 150 r.p.m., right handed, and carries a 96-tooth gear which drives a 24-tooth gear on a second shaft. A 72-tooth gear on the second shaft drives a 24-tooth gear on a third shaft. Find the speed of each shaft and make a sketch showing the arrangement of gears and directions of rotation produced.

14. A simple gear train has four gears with the number of teeth as follows: 96, 24, 48, and 60. If the last gear is on a shaft making 500 r.p.m. and turns right handed, find the speed and direction of rotation of the first gear.

15. According to Fig. 86, in what direction will gear 2 turn?

Find the speed ratio of the gears.



16. What is the direction of rotation of gear 2 according to Fig. 87? Find the number of teeth in gear 2.

17. Fig. 88 shows a train of simple gears. In what direction will gear 2 turn? Find the speed ratio of gears 1 and 2.

18. Fig. 89 shows a train of compound gears. In what direction will gear 4 turn.

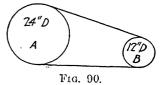
CHAPTER IX

BELTS AND PULLEYS. EFFICIENCY OF MACHINES

30. Belts and Pulleys.—Belts and pulleys are an important part of shop equipment and it is necessary to understand how to calculate speeds and diameters of pulleys and also lengths of belting.

Pulleys are nothing but gears without teeth and instead of running together directly they are made to drive one another by cords, ropes, cables, or belting of some kind.

As with gears, the speeds of pulleys are inversely proportional to their diameters. That is, if, as in Fig. 90, we have two pulleys A and B which are 24 in. and 12 in. in



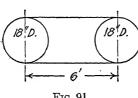
diam. respectively, the *larger* pulley travels the *slower* and the speed of A is to the speed of B as 12 is to 24.

This difference between gears running together directly and pulleys driven through belting should be fully understood. A belt "creeps" and "slips" and does not run the driver quite as fast as would be figured out from the diameters of the pulleys, while gears cannot slip and the practical speed and calculated speeds are the same.

When two pulleys are driven through a belt if there is no slipping of the belt, the rim speed of both pulleys is

the same and in Fig. 90 when pulley A makes 1 turn the belt passes over each pulley a distance equal to the circumference of pulley A or $2\pi 12$ or 24π in. and if the rim of pulley B has passed through 24π in. for 1 turn of pulley A and its diameter is 12 in. it has made a number of turns in this time equal to $\frac{24\pi}{12\pi}$ or 2. The following simple rule is convenient for calculating speeds of pulleys.

The diameter of the driving pulley multiplied by its



Frg. 91.

speed is equal to the diameter of the driven pulley multiplied by its speed. In calculating the length of belt to connect two pulleys if the pulleys are the same size and the belting is not crossed the length of belt required is equal to twice

the distance between the centers of the pulleys plus the circumference of one of the pulleys. In Fig. 91 the length of belt needed equals

$$2 \times 6 + \pi \times \frac{18}{12}$$

12 + 4.7 or 16.7 ft.

or

If the diameters of the pulleys are not the same, but do not differ greatly and the distance between their centers is large compared with the pulley diameters, the length of belt is found very nearly by adding to twice the distance between the pulley centers half the circumference of each of the pulleys.

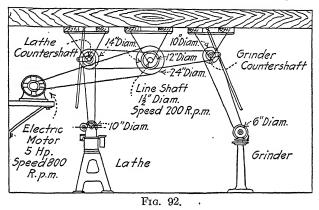
PROBLEMS

1. It is desired to run a countershaft at 200 r.p.m. from a line shaft running at 800 r.p.m. If the pulley on the line shaft is 18 in. in diam., what size pulley is required on the countershaft? 2. What length of belt is required to connect two 24-in. pulleys whose centers are 6 ft. 4 in. apart?

3. A belt runs from a 36-in. pulley on a line shaft to a second shaft having a 14-in. pulley. If the line shaft makes 650 r.p.m. what is the speed of the second shaft?

4. Find the length of belt required to connect a 24-in. and a 36-in. pulley whose centers are 8 ft. apart.

5. A main shaft running at 150 r.p.m. with an 18-in. pulley drives a jack shaft through a 12-in. pulley. A second pulley of 24 in. diam. on the jack shaft drives a dynamo through a 12-



in. pulley. Find the speed of the dynamo and make a sketch showing the arrangement of pulleys, and direction of rotation of shafts.

6. Calculate from the following rule the length of belting required to connect two 24-in. pulleys 7 ft. apart if the belt is crossed?

$$l = 2\sqrt{D^2 + S^2 + \pi D}$$

When D = diam. of pulleys

S = distance between centers

7. A main shaft running at 200 r.p.m. with a 12-in. pulley drives a second shaft through a 24-in. pulley. The slip of the belt is 2 per cent. of the driving speed. On the second shaft is a second pulley 36 in. in diam. which is connected through a belt to an 18-in. pulley on an electric generator. The belt on the generator

slips 3 per cent. of the driving speed of the second shaft. What is the speed of the generator?

8. Fig. 92 shows the elevation of a layout for a small machine shop, with the equipment as indicated. Working from this sketch, find

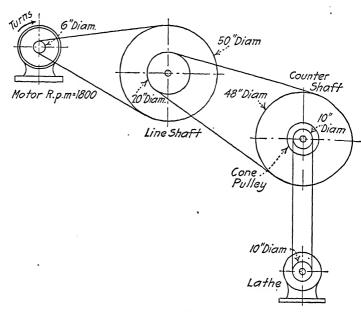


Fig. 93.

- 1. The diameter of pulley on motor shaft.
- 2. The speed in r.p.m. of lathe countershaft.
- 3. The speed of lathe. Pulley on lathe countershaft is 12 in . in diameter.
- 4. The speed of grinder countershaft. Pulley on grinder countershaft is 8 in. in diameter.
- 5. The speed of grinder.
- 6. The weight in pounds of line shaft. Length 10 ft., diam. 2½ in. Material, steel weighing .28 lb. per cu. in.

- 9. With the belts as arranged in Fig. 93, show the direction of turning of the lathe spindle, with the direction of rotation shown for motor. How could the direction of turning of the lathe be reversed? With data given on sketch what is the speed in r.p.m. of the lathe spindle when belted as shown? Why is it considered better practice to have the tight side of the belt on the bottom of the pulleys?
- 31. Efficiency of Machines.—In working out the problems on levers, pulleys, inclined plane and so forth we have not taken account of friction or other sources of loss. In other words we have supposed them to be perfect. As actually used their operation is not as satisfactory as our answers show, and as an indicator of the performance of a machine we often find its "efficiency."

Efficiency means $\frac{\text{the output of a machine}}{\text{input to the machine}}$ or better efficiency = $\frac{\text{output}}{\text{input}}$.

The efficiency is a number which shows how much of the energy the machine takes, it puts out again as useful work.

Suppose a screw jack has an efficiency of 20 per cent. or .20. This means that it delivers in useful work 20 per cent. of all the energy or work put into it.

If a machine had an efficiency of 100 per cent. or 1 it would mean that efficiency = $1 = \frac{\text{output}}{\text{input}}$ therefore input \times 1 = output, or output = input. But no machine has an efficiency of 100 per cent. and therefore the output can never equal the input since there are always some losses in the machine itself.

Suppose a 50-h.p. motor has an efficiency of 85 per

cent. when delivering its rated (50) horsepower. T_0 find its input we have

Efficiency =
$$\frac{\text{output}}{\text{input}}$$
, therefore input = $\frac{\text{output}}{\text{efficiency}}$ or input = $\frac{50}{.85}$ or 58.8 h.p.

If a steam engine delivers 50 h.p. and its input is 70 h.p., its efficiency is found as follows:

Eff. =
$$\frac{\text{output}}{\text{input}} = \frac{50}{70} = .714$$

.714 × 100 = 71.4 per cent.

or

It is therefore to be remembered that in working out a problem on machines we always have to take account of losses that are due to friction in the machines themselves in order to get the correct results.

PROBLEMS

- 10. An electric generator delivers 48 h.p. and requires 60 h.p. to drive it. What is its efficiency?
- 11. A screw jack lifts a 20-ton weight 2 ft. thereby doing $20 \times 2000 \times 2$ or 80,000 ft.-lb. of work, and in doing this work requires 334,000 ft.-lb. of work. Find its efficiency.
- 12. An engine is rated at 100 h.p. and has an efficiency at this load of 75 per cent. What is its input?
- 13. An electric motor takes 20 h.p. to do a certain amount of work. If its efficiency is 88 per cent., find its horsepower output.

CHAPTER X

MOTION

32. Definitions.—Motion is a change of position and may be of a number of kinds.

In order to set objects in motion forces or pressures are necessary. If a steady force acts on an object moving it, the object increases in motion until the resistance with which the object meets is equal to the force causing the motion and the object then travels over equal distances in equal times or as we say, has a constant speed. The word constant means unchanging, and speed, or what is the same thing, velocity, means the rate of motion of an object. If a train going at a steady rate covers fifteen miles in an hour its speed or velocity is 15 miles per hour (abbreviated m.p.h.). When an object is moving at a constant speed, if the driving force is removed, the speed decreases until the object is brought to rest by resisting forces such as friction, etc. When an object increases in speed it is said to "accelerate," and when it decreases in speed it is said to "retard."

Any object at rest will remain at rest for all time and an object in motion will always move at a constant speed in a straight line unless it is acted upon by a force or pressure which deflects it or changes its rate of motion, and how much an object will change in speed or direction or both depends on how large the force is acting on it.

When an object is free to fall the force of attraction of

the earth for it or the force of gravity causes the object to gain in speed 32.2 ft. per sec. each second of its fall.

33. Composition of Motions.—Motions, like forces, may be conveniently represented by straight lines. The length of the line representing to scale the rate of the motion as 10 ft. per sec. or 40 m.p.h. and the direction of the line, the direction of the motion. In Fig. 94 an object at 0 is represented as having two successive motions of 10 and 15 miles per hour, these motions being 60° apart. The effect of the two motions is the same

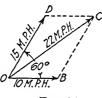


Fig. 94.

as one motion of 22 m.p.h. in the direction OC. This occurs, for example, if from a train traveling in the direction OB at 10 m.p.h., an object is thrown off in the direction OD at a speed of 15 m.p.h., the object itself is affected by both motions at the same time and travels in the direction

OC at the rate of 22 m.p.h. If two trains are traveling in the *same* direction, one at 40 m.p.h. and the other at 35 m.p.h. the *relative* speed of the trains is only 5 m.p.h.

If an engine hauls a freight train at the rate of 20 m.p.h. and a brakeman runs along the train in the direction of motion at a speed of 5 m.p.h. he is traveling at the rate of 20 + 5 or 25 m.p.h. with respect to the earth or track.

The following rules are convenient in determining speeds.

Velocity or speed = $\frac{\text{distance}}{\text{time}}$ and $\text{distance} = \text{speed} \times \text{time}$

Acceleration or rate of change of speed = $\frac{\text{speed}}{\text{time}}$

If a train starts from rest and increases uniformly in speed to 24 m.p.h. in 3 min. it has changed in speed or "accelerated" $\frac{24}{3}$ or 8 m.p.h. each minute. If this constant rate of increase of speed were kept up for 5 min. by a locomotive starting from rest, its speed at the end of the 5 min. would be 8×5 or 40 m.p.h.

It has already been stated that the earth attracting a freely falling object increases its speed about 32.2 ft. per sec. for each second of its fall.

PROBLEMS

- 1. A locomotive went from Altoona, Pa. to Harrisburg, Pa., a distance of 131 miles in 112 min. What was its average speed in miles per hr., and in ft. per sec.?
- 2. The same locomotive mentioned in Prob. 1 went from Altoona to Philadelphia, a distance of 236 miles in 3 hr. and 19 min. Making allowance for a 3-min. stop at Harrisburg, find the average speed made in miles per hr. and in ft. per sec.
- **3.** A passenger locomotive leaving a station attains a speed of 40 m.p.h. in 4 min. 30 sec. What is the acceleration or rate of change of speed?
- 4. An engineer has to make, in 3 hr., a 120-mile run with four 3-min. stops. What average rate of speed does he have to maintain in order to complete the run on time?
- 5. A passenger train making an average speed of 38 m.p.h. has to make a run of 85 miles. How much sooner will it complete the run than a freight which starts at the same time and makes an average of 25 m.p.h.?
- 6. The "Pennsylvania Limited" traveling at 60 m.p.h. starts from a station an hour after an accommodation train. The accommodation is delayed ½ hr. and is overtaken by the "Limited in 2 hr. What is the rate of speed of the accommodation train and how far had it traveled when overtaken by the "Limited?"
- 7. If the 82-in. drivers on a passenger locomotive make 200 revolutions per minute (r.p.m.) what is its speed in miles per hr.? What would be its speed allowing for a 2 per cent. slip?

- 8. A passenger locomotive with 80-in. drivers making 180 r.p.m. and a freight engine with 60-in. drivers making 150 r.p.m. are traveling in the same direction. What is the relative speed of the locomotives? If the locomotives were traveling in opposite directions what would be their relative speeds?
- 9. An engineer makes an average of 42 m.p.h. and runs for 4 hr. and 20 min. What distance does he cover?
- 10. If an object falls freely from rest for 3 sec., what is its speed in ft. per sec. and in miles per hr. at the end of that time?
- 11. The drivers on a locomotive are 18.85 ft. in circumference and make 250 turns a min. What is the speed of the locomotive in miles per hr.?
- 12. Two trains start from a station at the same time and travel over level country, the first North at the average rate of 40 m.p.h. and the second East at the average rate of 30 m.p.h. How far apart are the trains at the end of 5 hr.?
- 13. A train makes a total distance of 236 miles, covering one-third of this distance at the rate of 45 m.p.h. and the remaining distance at the rate of 40 m.p.h. In what time does it make the trip if its total stops take 10 min.?
- 14. A touring car starts from a garage and reaches a speed of 48 m.p.h. in 4 min. and 20 sec. What is its acceleration in miles per hr. per min., and in miles per hr. per sec.?
- 15. Find the speed in ft. per sec. and in miles per hr. of a freely falling object at the end of the fourth second of its fall.
- 16. In a trolley car the gear on the driving axle and the gear on the motor axle have a ratio of teeth of 4:1. The driving wheel has a diameter of 32 in. Find the speed of the trolley car in miles per hr., when the motor is making 1000 r.p.m.
- 17. Two trains start from a station at the same time, one going East at 40 m.p.h. for 5 hr. and the other North at 20 m.p.h. for 2 hr. and then East at 10 m.p.h. for 3 hr. Find the distance apart of the trains at the end of the 5 hr.

CHAPTER XI

CUTTING SPEEDS. SPEEDS OF LATHES

34. Definitions and Problems.—The calculation of the circumference and diameter of circles becomes necessary in calculating the cutting speed of work being turned off in a lathe or in calculating the rim speed of fly-wheels, grindstones, etc.

Machines, such as lathes, milling machines, boring mills, planers, etc., are provided with means for changing the speed according to the work turned out. Every mechanic should know how to calculate the proper size of pulleys or gears for any particular job. When a piece of metal of circular section is being turned down in a lathe the cutting tool for each revolution of the work travels a distance (with respect to the metal being turned down) equal to the circumference of the work.

For example, a rod 2 in. in diam. passes a distance of 3.142×2 or 6.284 in. under the cutting tool for each revolution of the rod. If the rod rotated 60 times per min. the cutting speed would be 60 times 6.28 or 376.8

in. per min. or $\frac{376.8}{12}$ or 31.4 ft. per min.

If S is the surface speed in ft. per min.,

N the revolutions per min. of the work,

C the circumference of the work in in.,

the rule is
$$S = \frac{C \times N}{12}$$

For example, a 2-in. diam. brass rod when turned at the rate of 160 r.p.m. has a surface speed of

$$S = \frac{2 \times 3.142 \times 160}{12}$$
 or 83.8 ft. per min.

The rim speed of a fly-wheel 72 in. in diam. when running at 200 r.p.m. may be found by the same rule as follows:

$$S = \frac{72 \times 3.142 \times 200}{12}$$
 or 3770 ft. per min.

Working the other way round we can find the revolutions per min. (N) at which a wheel must rotate to have a given surface or rim speed.

If a fly-wheel 48 in. in diam. is to have a rim speed of a mile a min. or 5280 ft. per min. its revolutions per min. are found from the rule

$$N = \frac{12 \times S}{C}$$
, that is Revolutions per min. $= \frac{12 \times \text{speed in ft. per min.}}{\text{circumference of wheel in in.}}$
Therefore $N = \frac{12 \times 5280}{48 \times 3.142} = 420 \text{ r.p.m.}$

There are average values given for cutting speeds of different metals as follows:

Cutting Speeds in F	t. per Min.
Brass	80
Machinery steel	30
Tool Steel	20
Cast Iron	40

With high-speed steel cutting tools about double the above speeds can be used. No exact figures can be given for the best cutting speeds of the different metals. The

proper speed to use depends on the size of the work, the power and rigidity of the lathe, the kind of tool used and the nature of the work being turned out.

For the surface speed of grindstones the following figures may be taken

For carpenters' tools, 600 ft. per min. For machinists' tools, 1000 ft. per min.

When edge tools are ground care must be taken to avoid too high a speed to prevent heating the edges of the tools and getting rough surfaces. For rapid grinding the following speeds are used:

With Huron stones, 3500 ft. per min. With Ohio stones, 2500 ft. per min.

Some polishing wheels are run as high as 8000 ft. per min. Emery wheels are run usually at a surface speed of about a mile a min. or 5280 ft. per min.

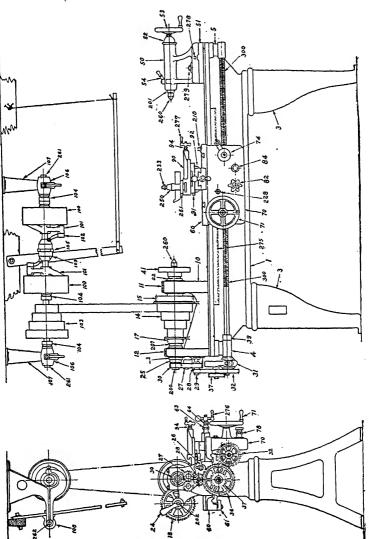
To find how long it will take the cutting tool on a lathe to travel a given distance along the work with a given feed we proceed as follows:

Suppose a lathe tool has a feed of $\frac{1}{8}$ in. per rev., and we wish to find how long it will take to turn a length of 18 in. on an 8-in. diam. piece with a cutting speed of 60 ft. per min. The circumference of the piece is 8×3.142 or 25.136 in. This is equal to $\frac{25.136}{12}$ or 2.094 ft. With a cut-

ting speed of 60 ft. per min. this requires $\frac{60}{2.094}$ or 28.6 r.p.m.

For each revolution with a feed of $\frac{1}{8}$ in, the tool travels along the work $\frac{1}{8}$ of an in, or $28.6 \times \frac{1}{8} = 3.57$ in, per min. To turn a length of 18 in, would therefore require

 $\frac{18}{3.57}$ or 5.04 min. or a little over 5 min.



Cut of Lathe showing principul paris numbered.

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NUMBER AND NAME OF LATHE PARTS ON DRAWING

NO.

1. Bed.

3. Power Legs.

4. Lead Screw Bracket F.

5. Lead Screw Bracket R.

10. Head Stock.

11. Head Stock Cap, Large.

12. Head Stock Cap, Small.

13. Head Stock Clamp Plate.

14. Spindle Cone.

15. Bull Gear.

16. Bull Gear Clamp.

17. Cone Pinion. 18. Quill Gear.

19. Quill Sleeve.

20. Quill Sleeve Pinion. 21. Ecc. Shaft Bushing.

22. Bronze Box, Large. 23. Bronze Box. Small.

24. Back Gear Lever.

25. Spindle Take up Nut.

26. Reverse Bracket.

27. Reverse Twin Gears (2).

28. Reverse Gear.

29. Stud Gear.

30. Spindle Reverse Gear.

31. Change Gear Bracket.

32. Change Gears.

33. Change Gear Idler.

34. Bushing for Change Gear Idler.

35. Change Gear Collar on L. S.

36. Compound Idler Gear 1 to 2 Large.

37. Compound Idler Gear 1 to 2

38. Bushing for Compound Idler

Gear, I to 2.

39. Thrust Collar on Lead Screw.

40. Large Face Plate.

41. Small Face Plate.

42. Turning Gear.

50. Tail Stock Top.

51. Tail Stock Base.

52. Tail Stock Nut. 53 Tail Stock Hand Wheel NO.

56. Tail Stock Clamp Plate.

60. Saddle.

61. Saddle Gib.

62. Saddle Lock.

63. Cross Feed Bushing.

64. Cross Feed Gra. Collar.

65. Cross Feed Nut.

66. Plain Rest.

67. Thread Cutting Stop.

70. Apron.

71. Apron Hand Wheel.

72. Lead Screw Half Nut.

73. Lead Screw Half Nut Gib (2).

74. Nut Cam.

75. Nut Cam Washer.

76. Rack Pinion Gear.

77. Auto. Apron Worm Wheel.

78. Auto. Apron Clutch Sleeve Bushing.

79. Auto. Apron Worm Bracket.

80. Auto. Apron Clutch Sleeve.

81. Auto. Apron Clutch.

82. Auto. Apron C. F. Star Knob.

83. Auto. Apron C. F. Lever.

84. Auto. Apron C. F. Lever Knob.

85. Auto, Apron C. F. Gear.

86. Auto. Apron Idler Gear.

87. Auto. Apron Idler Gear Pinion.

90. Compound Rest Top.

91. Compound Rest Swivel.

92. Compound Rest Bottom.

93. Compound Rest End Cap.

94. Compound Rest Bushing.

95. Clupound Rest Nut.

96. Compound Rest Chip Guard.

Countershaft

100. C. S. Friction Pulleys (2).

101. C. S. Friction Spiders (2).

102. C. S. Friction Fingers (2).

103. C. S. Cone.

104. C. S. Collars (4).

105 C S Voke Lever

NO.

108. C. S. Shipper Nut.

109. C. S. Yoke Cone.

200. Head Stock Spindle.

201. Tail Stock Spindle.

202. Back Gear Eccentric Shaft.

203. Apron Worm.

204. Apron Rack Pinion.

205. Spindle Sleeve.

207. Spindle Thrust Collar.

208. Apron Worm Collar.

209. Tool Post Block.

210. Carriage Lock Collar Screw.

211. Compound Rest Swivel Bolts.

212. C. G. Bracket Collar Screw.

213. Reverse Collar Screw.

214. Bull Gear Clamp Collar Screw.

215. Apron Clutch Sleeve Hex. Nut.

216. Compound Rest Swivel Stud.

217. Steady Rest Lock Bolt.

218. Auto Cross Feed Lever Stud.

219. Reverse Steel Washer 1/2" hole.

220. Apron Clutch Sleeve Pinion.

221. Compound Rest Bottom Gib.

222. Plain Rest Gib.

223. Auto Apron Clutch Screw.

224. Cross Feed Screw.

225. Apron Hand Wheel Pinion.

226. Tail Stock Screw.

227. Reverse Shaft or Stud.

228. Apron Rack Pinion Stud.

229. Reverse Shoulder Screws (2).

NO.

230. Compound Rest Screw.

231. Auto Cross Feed Stud.

232. Apron Half Nut Stud (2).

233. Tool Post Screw.

234. Apron Idler Gear Stud.

235. Cam Cap Screw.

238. Apron Worm Washer.

239. Comp. Rest Steel Wedge.

240. Gap Bridge Pins (2).

241. Reverse Stud Collar 34" hole.

242. Change Gear Spindle Knob.

250. Tool Post.

251. Tool Post Ring.

252. Tool Post Wedge.

253. Tool Post Wrench.

254. Compound Rest Wrench.

257. Compound Rest Top Gib.

258. Comp. Rest C. P. Headless Set Screws.

260. Centers (2).

261. C. S. Shaft.

262. C. S. Shipper Rod.

263. C S. Expansion Wedges.

275. Rack.

276. Cross Feed Ball Crank.

277. Compound Rest Handle.

278. Tail Stock Set Over Screws (2).

279. Tail Stock Clamping Bolt, Nut and Washer.

300. Lead Screw.

PROBLEMS

1. A steel bar 3 in. in diam is to be turned in a lathe. If the cutting speed is to be 30 ft. per min. at how many revolutions per min. should the bar turn?

2. A cast-iron pulley 18 in. in diam. is to have its rim face turned down in a lathe. With a cutting speed of 40 ft. per min. what should be the revolutions per min. of the pulley?

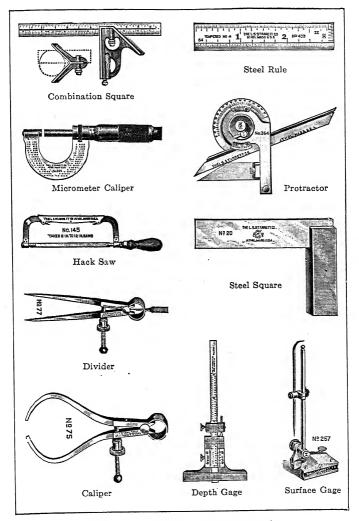


Fig. 96 —Cut showing machinist tools.

3. An emery wheel 14 in. in diam. runs at 1380 revolutions per minute (r.p.m.). Find its surface speed in ft. per min.

4. What is the speed of a belt running on a 3-ft. pulley which

makes 300 r.p.m.?

5. At how many revolutions per min. should we run a 2-in. diam. brass rod in a lathe for a cutting speed of 80 ft. per min.?

6. Find the revolutions per min. for a $1\frac{1}{8}$ -in high-speed drill to give a cutting speed at the outer edge of 80 ft. per min. If the feed is .015 in. per rev., how long will it take to drill through $1\frac{1}{2}$ in. of metal?

7. Find the revolutions per min. of an 18-in. polishing wheel whose surface speed is 8000 ft. per min.

8. At how many revolutions per min. should a 30-in. grindstone be run to obtain the proper surface speed for grinding machinists' tools?

9. A piece of work 6 in. in diam. is being turned down at a cutting speed of 60 ft. per min. (f.p.m.). If the cutting tool feeds along the work ½ in. for each revolution, how long will it take to turn a length of 8 in.?

10. A steel crank shaft is to be finished to a diameter of 8 in. With a cutting speed of 40 f.p.m. what should be its speed in revolutions per min.?

CHAPTER XII

VOLUME AND PRESSURE OF GASES

35. Definitions and Problems.—A very important law has been discovered in regard to the relation of the volume, temperature and pressure of gases. Steam follows this law very closely. It has been found that if the temperature of a gas does not change, its volume is practically "inversely proportional" to its pressure, or its volume times its pressure is a constant quantity. This means that if we have a cylinder, as in Fig. 97, which contains

10 cu. ft. of air with the piston in position A and the gauge reads 10 lb., making an absolute pressure of 10 + atmospheric pressure (that is, 10 + 14.7 or 24.7 lb.), and we move the piston to the posi-

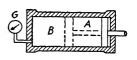


Fig. 97.

tion B so that the volume is only 5 cu. ft. or one-half of its former value, the absolute pressure becomes 2×24.7 lb. or 49.4 lb. or twice its former value and the gauge reads 49.4-14.7 or 34.7 lb. since the gauge is made to read pressures above that of the atmosphere. That is, making the gas occupy half the volume has doubled its pressure. If, on the other hand, we increase the volume of the gas, we lower its pressure. This is exactly what happens in a steam engine cylinder into which steam enters at perhaps 200-lb. pressure, then expands, does work

in moving the piston, becomes reduced to a comparatively low pressure and exhausts into the air.

The principle of expansion and compression of gases is made extensive use of in all steam and gas engines and in air-braking devices. The rule is: If the temperature of a gas does not change, the product of its volume and pressure remain practically the same throughout a considerable range of pressure.

PROBLEMS

1. If the air in a cylinder containing 5 cu. ft. at atmospheric pressure is compressed to a volume of 1 cu. ft., what is its pressure?

2. A gas occupies 10 cu. ft. at an atmospheric pressure of 15 lb. per sq. in. and is afterward compressed to a volume of 2 cu. ft. What is its pressure?

3. During the movement of a piston in a steam cylinder the pressure drops from 90 lb. to 50 lb. If this change took place according to the law explained, what change of volume took place?

4. A 30-in. × 36-in. cylinder with a .2 cu. ft. clearance volume at each end takes in air at atmospheric pressure of 14.7 lb. What is the pressure of this air when the piston has completed one-half the return stroke and is compressing the free air drawn in on the forward stroke?

5. Twenty cubic feet of free air is compressed to a pressure of 60 lb. gauge. What volume does it then occupy?

6. The air cylinder of an air compressor is 20 in. × 28 in. and its clearance volume is .2 of a cu. ft. What is the pressure of the discharged air if the outlet valves lift at ¾ stroke?

7. If the admission of steam to a 24-in. × 26-in. cylinder at a pressure of 200 lb. per sq. in., gauge is cut off at 50 per cent. of the piston stroke (½ stroke) what is the terminal pressure, that is, the pressure at the end of the stroke?. (Give the answer in gauge pressure.) Clearance volume at each end of cylinder is .2 cu. ft.

8. What would be the terminal pressure if the cut-off occurs at 25 per cent. of the stroke (1/4 stroke) with the conditions of Prob. 7?

- 9. With a back pressure on the piston during the exhaust stroke of 3 lb. per sq. in. gauge, what would be the compression pressure (lb. per sq. in. gauge) at the end of the stroke when compression or exhaust closure occurs at .6 of the piston stroke, in a 24-in. × 26-in. cylinder? Clearance volume at end of cylinder is .2 cu. ft.
- 10. What would be the compression pressure (lb. per sq. in. gauge) if compression of the steam occurs at % stroke with other conditions the same as in Prob. 9?

CHAPTER XIII

WORK AND POWER.

36. Definitions and Rules on Work.—Work means the overcoming of resistance of any kind. If you drag an object along the floor you do work in overcoming the friction between the object and the floor. In lifting an object you do work against gravity which tends to pull the object toward the earth. Steam in a locomotive cylinder does work when it expands and moves the piston against the resisting forces. "Work" is measured in "footpounds," that is, the product of the resistance overcome in lb. and the distance in ft. through which it is overcome. If you lift a 10-lb. weight 5 ft. you do 10×5 or 50 ft.-lb. of work against gravity.

If a train weighs 350 tons and the resistance offered to its motion is 12 lb. per ton, the work done in moving the train 1 mile at uniform speed is

$$350 \times 12 \times 5280 = 22,176,000$$
 ft.-lb.

If it takes an average pressure of 30 tons to punch a $\frac{3}{6}$ -in. steel plate, the work done equals the resistance in lb., or 30×2000 lb. times the thickness of the plate in ft., $\frac{3}{6}$ in. = $\frac{1}{3}$ 2 ft.

Therefore work done =
$$30 \times 2000 \times \frac{1}{32}$$

= 1875 ft.-lb.

37. Power, Horsepower.—Power means the rate at which work is done and is expressed in foot-pounds per

or

min. or in foot-pounds per sec. When 550 ft.-lb. of work are done in a sec. or 33,000 ft.-lb. in a min. 1 h.p. is represented. A 10-horsepower (abbreviated h.p.), engine therefore is one which can do $33,000 \times 10$ or 330,000 ft.-lb. of work in 1 min., or is one capable of overcoming a resistance of 330,000 lb. through a distance of 1 ft. in 1 min.

The rule for horsepower is:

Horsepower = $\frac{\text{pounds} \times \text{feet}}{550 \times \text{seconds}}$ Horsepower = $\frac{\text{pounds} \times \text{feet}}{33,000 \times \text{minutes}}$

In electrical terms 1 h.p. is equal to 746 watts. The horsepower required in any case depends as much on the *time* in which the work is to be done as on the *amount* of work to be done.

PROBLEMS

1. What is the horsepower of an engine that will raise 8000 lb. of water 200 ft. in 5 min.?

2. What is the horsepower of an automobile engine which can do 82,500,000 ft.-lb. of work in 5 min.?

3. What horsepower motors are required to raise a 222,000-lb. locomotive 4 ft. in 1 min.?

4. If a man in a day of 8 hr. raises 20,000 lb. of gravel through an average distance of 5 ft., what horsepower represents his working power?

5. A man weighing 150 lb. runs up stairs to a vertical height of 20 ft. in 2 min. What horsepower is he exerting to raise his own weight?

6. What is the useful horsepower of an engine which draws a train of 200 tons on a level at the uniform rate of 30 miles per hr. against a resistance of 16 lb. per ton weight?

7. A 7-h.p. engine is used to raise a weight of 24 tons. How high will it raise it in 1 min.? If it is desired to raise the weight in twice the time, what horsepower is required?

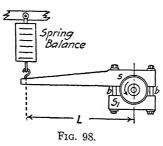
- 8. It is desired to raise a weight of 5 tons 200 ft. in 40 min. What horsepower is necessary to do the work?
- 9. What average horsepower is required merely to lift a 250-ton train up 5 miles of grade rising 1 ft. in every 528 ft., if the train is moving at the rate of 25 miles per hr.?
- 10. If the name plate on a motor gives the motor rating as 5 h.p., how many foot-pounds of work should the motor do per min. without overheating? Can the motor be worked above its rated horsepower?
- 11. From the rule that horsepower per cylinder = $\frac{PLAN}{550}$ in which P is the average steam pressure in pounds per sq. in., L the length of stroke in ft., A the area of the piston in sq. in., and N the number of strokes per sec., find the horsepower developed on both sides of a locomotive whose cylinders are 20 in. \times 28 in., working with an average steam pressure of 180 lb. per stroke and 150 strokes per min.?
- 12. A 12-ton casting is raised 20 ft. in 1 min. by a traveling crane. What horsepower does the motor on the crane exert?
- 13. From the rule for horsepower per cylinder, h.p. = $\frac{PLAN}{33,000}$ in which P is the average pressure in pounds per sq. in. on the piston, L the length of stroke in feet, A the area of piston in square inches, and N the strokes per min. (or 2 times the r.p.m.) find the horsepower for both cylinders of the following locomotive. Cylinders 24 in. \times 26 in. (24 in. diam. of piston and 26 in. stroke of piston), average steam pressure 160 lb. per sq. in., and running at 200 r.p.m.
- 14. From the rule in the foregoing problem calculate the total cylinder horsepower of a 4-cylinder Mallet locomotive.

Cylinders 27 in. × 28 in., average steam pressure on piston 130 lb. per sq. in., and r.p.m. 150.

15. Find the horsepower of a freight locomotive running at 10 m.p.h. with an average cylinder pressure of $\frac{1}{2}$ boiler pressure. Drivers 62 in. in diam., cylinders 24 in. \times 28 in., boiler pressure 205 lb. per sq. in.

The power delivered by an engine or a motor at its pulley, that is, the actual power available for doing work may be found for small machines by the Prony-Brake Method.

Fig. 98 shows such a brake consisting of shoes s and s_1 , which are clamped to the machine pulley by bolts b and b, which may be regulated to increase or decrease the pressure of the shoes on the pulley. When the pulley rotates in the direction of the arrow, the entire brake and lever L tend to rotate in the same direction. This tendency is counterbalanced by the pull on the spring shown, from which we may read the pounds pull for different degrees of pressure of brake shoes on pulley.



The spring keeps the lever arm practically horizontal throughout the test. The power absorbed by the brake shoes "s" and "s₁" equals the amount of work done by the revolving pulley on the machine. In using the brake we first take the reading of the

spring balance with the machine not running in order to account for the weight of the lever itself, then after bringing the machine up to speed and putting a "load" on it by means of bolts b and b, the balance reading is again taken, and the initial reading subtracted to get the net pull due to the rotation alone. If P is the pull in pounds on the balance, L the length of the lever as shown and r.p.m. the revolutions per min. of the pulley, the foot-pounds of work done per min. equals $P \times 2\pi L \times r.p.m.$, and the horsepower equals

$$\frac{P \times 2\pi L \times \text{r.p.m.}}{33,000}$$
.

When used in tests of this kind the machine pulley is made so that water may be run on the inside of the rim to keep the pulley cool and absorb the heat produced by the transformation of the mechanical work done at the pulley.

The principle of the brake described above is the same as that of the power of a belt in which the effective belt pull is "P" pounds on a pulley of L radius and running at a speed represented by the symbol r.p.m. In this case the belt moves at a speed of $2\pi L \times \text{r.p.m.}$ feet per min. and the horsepower is therefore $\frac{P \times 2\pi L \times \text{r.p.m.}}{22,000}$ or

the same rule as derived above.

The principle of the brake may also be considered the same as that of a weight "P" being raised by a rope wound on a pulley of radius L, as in Fig. 99.

Example.—Find the horsepower delivered by a direct-current motor in a brake test in which the net effective pull at end of lever $2\frac{1}{2}$ ft. long is 10 lb., with the pulley rotating at 900 r.p.m.

$$\begin{array}{c} -L \rightarrow \\ \hline P \end{array}$$

Fig. 99.

Solution.—Horsepower = $\frac{P \times 2\pi L \times \text{r.p.m.}}{33,000}$ or $\frac{10 \times 2 \times 3.14 \times 2.5 \times 900}{33,000}$

which equals 4.282 or practically 4.28 h.p.

PROBLEMS

- 16. Find the horsepower of an engine whose shaft rotating at 90 r.p.m. produces a pull of 1800 lb. at the end of a brake arm 9 ft. long.
 - 17. In a brake test on a motor it was found that with a speed

of 750 r.p.m., a pull of 20 lb. was produced at the end of a 5-ft. lever. What horsepower did the motor deliver?

To find the pull produced at the end of a given lever at a given speed and horsepower we have by transposing the horsepower rule,

$$P = \frac{\text{horsepower} \times 33,000}{2\pi L \times \text{r.p.m.}}$$

Writing this equation for the length of lever arm "L" we have

$$L = \frac{\text{horsepower} \times 33,000}{P \times 2\pi \times \text{r.p.m.}}$$

and for the revolutions per min., we have

Revolutions per min. =
$$\frac{\text{horsepower} \times 33,000}{P \times 2\pi L}$$

These rules enable us to find any one of the variable quantities in the rule in terms of the other quantities.

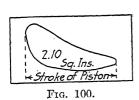
PROBLEMS

- 18. What pull "P" is produced on the end of a 4-ft. brake lever when the machine pulley is making 250 r.p.m. and machine is delivering 8 h.p.?
- 19. Find the brake horsepower of a steam engine running at 250 r.p.m. and exerting a pull of 80 lb. at the end of a lever 6 ft. long?
- 20. What length of lever arm is required for a pull of 18 lb., with a speed of 180 r.p.m. and a load of 2 h.p.?

The brake horsepower is the power of an engine or motor or water wheel available for use at the machine pulley and does not include the power lost in the machine itself through friction.

If in a steam engine we found the horsepower developed in the *cylinder* and called the *cylinder* or *indicated* horsepower we would find that this power is greater than the *brake* horsepower by the amount of power lost in friction in the engine mechanism between the cylinder and the fly-wheel or pulley. The horsepower thus lost is called the *frictional horsepower*.

In finding the *indicated* or cylinder horsepower of an engine we use a steam indicator or as it may be called, a recording steam gauge which takes a record of the steam pressure in the cylinder throughout the stroke of the piston. Such a card diagram or record is shown in Fig. 100. In making a test indicator cards are taken at each end of the cylinder. From these we find the average pressure throughout the stroke by finding the area of the card, dividing this by the length of the card, and multiplying by the constant of the indicator. For



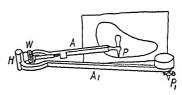


Fig. 101.

example, if the eard shown has an area of 2.10 sq. in. and its length is 3.73 in., the average height $= \frac{2.10}{3.73}$ or .563 in. If the scale of the diagram is 80 lb. to the inch this represents an average pressure of .563 \times 80 or 45 lb. per sq. in.

From this average pressure called the mean effective pressure we calculate the horsepower from the rule

Horsepower =
$$\frac{PLAN}{33,000}$$
.

In which P is the mean effective pressure. L is the length of stroke in ft. A is the area of the piston in sq. in. N is the number of strokes per min.

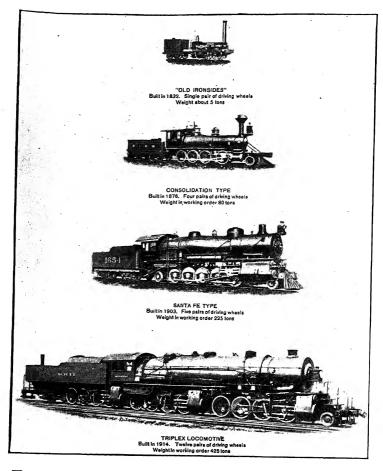


Fig. 102.—Cut showing the development in size and weight of locomotives.

To find the area of the indicator card shown, in Fig. 100 we make use of a planimeter or plane meter as shown in Fig. 101. This consists of the two arms A and A_1 hinged at H and with pin points at P and P_1 . The point P_1 is fixed and the point P is moved around the outline of the card as shown, while the wheel W moves and carries a scale which indicates directly the number of square inches in the area traced around when the point P has completed the outline.

CHAPTER XIV

CALCULATION OF BELTING. ENERGY

38. Horsepower of Belting.—When two shafts are too far apart to connect them by gears for transmitting power, belts are used. The power which a belt transmits depends on the effective pull tending to turn the driven pulley and the speed with which the belt is traveling. The pull which a belt can safely stand depends on its material, width and thickness. They are sometimes made of one, two, three or four thicknesses of leather and are called single, double, triple or quadruple. The widths of belts vary from about an inch to 2 ft. depending on the amount of power transmitted. Whether single, double or other thickness of belting is used depends largely on the size of pulleys on which the belt runs. Practice shows that single belts are best on pulleys less than a foot in diameter, and double belts on pulleys a foot or larger in diameter. On main drives or in cases where the amount of power transmitted would cause an excessively wide double belt a triple or quadruple belt is used.

It has been found that single leather belts will operate satisfactorily with an allowable pull of about 30 lb. per in. of width. For a double belt about 65 or 70 lb. is allowed per in. of width. Almost any mechanical handbook contains figures taken from the results of practice for the best allowable sizes of belts.

Practice shows that the linear speed of belts should not exceed 4500 ft. per min. if the belt is to make good contact with the pulleys. As usually operated, speeds below this value are used.

If P is the allowable pull per in. of width of belt, W the width of belt in in., the pull times the width, that is $P \times W$, gives the total effective pull or the force transmitted by the belt. This force in lb. multiplied by the speed of the belt in ft. per min. gives the footpounds of work per min. transmitted by the belt.

To obtain horsepower we divide foot-pounds per min. by 33,000, since 1 h.p. is the performance of 33,000 ft.-lb. of work per min. If S is the speed of the belt in ft. per min: and using the letter given above for the effective pull on the belt, we have the rule

Horsepower transmitted by belt =
$$\frac{P \times W \times S}{33,000}$$
.

Example.—Find the horsepower transmitted by a belt 5 in. in width, if the allowable pull per in. of width is 35 lb. and the speed of the belt 2400 ft. per min.

Solution:

Horsepower =
$$\frac{P \times W \times S}{33,000}$$
 or $\frac{35 \times 5 \times 2400}{33,000}$

which equals 12.7.

To find the width of belt to transmit a given horsepower for a given belt speed, we transform the above equation to express W in terms of the other quantities involved as follows.

If H represents the horsepower transmitted we have as above, $H = \frac{P \times W \times S}{33,000}$ and therefore $W = \frac{H \times 33,000}{P \times S}$.

Example.—Find the width of double leather belt re-

quired to transmit 100 h.p., if the speed of belt is 4000 ft. per min. and the allowable pull per in. of width is taken as 70 lb. since this is a double belt.

Solution:

$$W = \frac{H \times 33,000}{P \times S} \text{ or } \frac{100 \times 33,000}{70 \times 4000} \text{ which equals } 11.8$$

in. Therefore a 12-in. belt should be used.

PROBLEMS

1. Find the width of a single belt required to transmit 95 h.p. when belt runs on a 14 ft. diam. pulley at 70 r.p.m. when P = 40 lb.

NOTE.—The speed of the belt in ft. per min. equals the product of the circumference of the pulley in ft. by the r.p.m. of the pulley.

- 2. Find the width of belt required to transmit 15 h.p., if belt speed is 3500 f.p.m. and allowable pull per in. of width of belt is 50 lb.
- 39. Definitions on Energy, and Problems.—Energy is power to do work.

All objects possess energy on account of work having been done upon them at some time. Energy, like work, is measured in foot-pounds. There are, in general, two kinds of energy due either to the motion or to the position of objects. Energy of motion is called kinetic energy and energy of position is potential energy. For example, an object set in motion can overcome a certain amount of resistance before being brought to rest, and the energy which the object has on account of its motion is used up in overcoming the resistance, bringing the object to rest. Fly-wheels on engines both receive and give up energy and thus cause the engine to run more smoothly throughout the stroke.

Elevated weights have power to do work on account

of their elevated position, as in various types of hammers, pile drivers, etc. The rule for finding the kinetic energy (K.E.) of an object is:

$$K.E. = \frac{Wv^2}{64.4}$$

In which W is the weight of the object in lb., v is its velocity or speed in ft. per sec. Thus a weight of 230 lb. moving with a speed of 1200 ft. per sec. has a kinetic energy of $\frac{230 \times 1200 \times 1200}{64.4}$ or approximately 5,140,000 ft.-lb. The rule for potential energy (P.E.) is:

$$P.E. = W \times H$$

W is the weight of the object in lb. and H is its height in ft. above a given point. The energy is then given in foot-pounds. For example: A 2-ton steam hammer when raised 4 ft. has a potential energy of $2 \times 2000 \times 4$ or 16,000 ft.-lb. on account of its position and if let fall it would do 16,000 ft.-lbs. of work.

PROBLEMS

- 3. A weight of ⅓ ton is moving with a speed or velocity of 5 ft. per sec. How much energy in foot-pounds is stored up in it?
- **4.** The rim of a fly-wheel weighing 3 tons moves with a speed of 20 ft. per sec. How many foot-pounds of energy are stored up in it?
- 5. A 3-ton hammer is raised and dropped a distance of 8 ft., 8 times per min. How much work does it do per min.?
- **6.** Three thousand gallons of water are pumped into a tank an average height of 10 ft, above the level of a track. What energy does this body of water have with respect to the track?
- 7. If a punch makes a hole through a ½-in, steel plate and meets with an average resistance of 30 tons, how much energy is consumed in the punching alone?
 - 8. A machinist wields a 14-lb, hammer and strikes with a veloc-

ity of 30 ft. per sec. and 28 blows per min. How many footpounds of energy are produced per min.? At what horsepower does he work?

9. How much work in foot-pounds is required to raise a 10-ton casting vertically through a height of 5 yd.?

10. If 10,000 cu. ft. of water flow per min. over a fall 20 ft. high, how many foot-pounds of energy does this water deliver at the foot of the fall? (1 cu. ft. of water weighs 62.5 lb.)

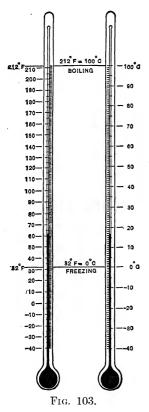
CHAPTER XV

HEAT

- 40. Definitions.—Heat is one form of energy. Other forms of energy such as electrical, chemical, energy of position and of motion can be changed into heat. Heat is produced by friction, by chemical action, and by the electric current flowing through a resistance such as copper or iron wire. In lighting a match heat is produced both by friction, and by chemical action in the burning of the match. The heat produced by a cutting tool on the metal being turned off in a lathe, and also that produced in grinding tools, are familiar examples of heat produced by friction, or in other words, by the change of mechanical work into heat. The burning of fuel is an example of a chemical change involving heat action. Objects in motion become heated when brought to rest by friction or by striking other objects.
- 41. Temperature.—The temperature of an object means not only whether the object is "hot" or "cold," but whether it takes heat from, or gives heat to, surrounding objects. When two objects at different temperatures are brought together there is a tendency toward an equalization of temperature. Like water which flows from a point of high to one of low level, heat flows from an object of high to one of low temperature. For measuring temperature, in most cases for ordinary work,

mercury thermometers are used, either the Fahrenheit or the Centigrade as shown in Fig. 103.

The Centigrade thermometer has 100 equal divisions on its scale between the point of melting ice and boiling



water, with the 0 mark at the point of melting ice.

The Fahrenheit thermometer has 180 equal divisions on its scale between the points where ice melts and water boils. The 32-degree mark is placed at the point where ice melts and the 212-degree mark at the point where water boils.

To change from a temperature in degrees Fahrenheit to the corresponding temperature in degrees Centigrade, and vice versa, the following rules are used.

F.° = C.°
$$\times \frac{9}{5} + 32$$
.
C.° = (F.° - 32) $\frac{5}{6}$

For example, using the first rule, suppose we wish to find the temperature Fahrenheit corresponding to 30° Cent., we have

F.° =
$$30^{\circ} \times \frac{9}{5} + 32$$

or F.° = $54 + 32$ or 86° .

That is, 86° on the Fahrenheit thermometer correspond to 30° on the Centigrade thermometer.

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Working the other way round and using the second rule

$$30 = (F. - 32)\frac{5}{9} \text{ or } 30 = \frac{5 F.}{9} - \frac{160}{9}$$

or $270 = 5 F. - 160 \text{ and } F. = 86^{\circ}.$

Alcohol and air thermometers are also used for measuring temperatures and these as well as the mercury thermometers make use of the principle of expansion and contraction of the alcohol, air or mercury with different temperatures, to obtain different readings.

42. Heat Measurement.—Heat is measured in *British Thermal Units* (abbreviated B.t.u.). One B.t.u. is the amount of heat required to raise the temperature of 1 lb. of water 1° Fahr.

For example, if 10 lb. of water are heated from 60° Fahr. to 90° Fahr. the amount of heat taken up by the water equals $10 \times (90^{\circ} - 60^{\circ})$ or 10×30 or 300 B.t.u.

When water reduces in temperature it of course gives up heat and the calculations are made in the same way by multiplying the weight of water in lb. by the drop in temperature in degrees Fahrenheit. This gives the number of B.t.u. liberated by the water. It should be understood that temperature means heat "intensity" or as we might say heat "pressure" and is measured in "degrees," while "quantity" of heat is measured in B.t.u. We can make the same comparison with air, in which the "intensity" or "pressure" is measured in "lb." by a gauge, while the "quantity" of air is measured in cu. ft. Again in the case of steam, we measure the "pressure" in "lb." and the "quantity" in lb. of condensed steam.

If two iron castings are of different weights, it will take

a different "quantity" of heat in B.t.u. to heat them to the same "intensity" or "temperature."

In the same way two air cylinders can contain different quantities of air and still both be at the same pressure.

If the change of temperature is given in Centigrade rather than in Fahrenheit degrees we multiply the number of Centigrade degrees by % since 1 Centigrade degree equals % Fahrenheit degrees.

If the weight is given in units other than pounds we first change the weight to the equivalent number of pounds.

Example.—How much heat in B.t.u. is taken up by 1 ton of water which is heated from 10° Cent. to 40° Cent.?

Solution.—B.t.u. =
$$1 \times 2000 \times (40-10) \times \frac{9}{5}$$
 or 2000

$$\times \frac{30 \times 9}{5}$$
 which equals 108,000. Ans.

43. Expansion and Contraction Due to Heat.—Most substances expand or increase in volume when heated and contract or decrease in volume when heat is removed from them.

This effect takes place in solids, liquids, and gases. The principle of expansion and contraction is made of considerable use and in some cases it may cause considerable trouble unless allowed for.

When a piece of metal, for example, is heated, it expands in all directions. The expansion in one direction or dimension is that with which we are particularly concerned and is called the "linear" expansion of the substance.

For example, in the case of mercury thermometers the principle of expansion is used, also in steam piping, HEAT 109

allowance must be made for the lengthening and shortening of the pipe system due to heat changes. In laying steel rails an allowance is made between the ends of the rails for contraction and expansion. The same is true in the case of bridges and other metal structures.

The power of expansion and contraction of metal is very great and if allowance is not made for it the expansion will take place just the same with the result that the part expanding will buckle or give itself the necessary room for expansion in one way or another. These characteristics of metals are made use of in putting tires on wheels, in making shrink fits and numerous other applications.

In the case of tires put on wheels and shrink fits of various kinds, an allowance is made per in. of diameter of the object on which the fit is to be made of approximately 11000 of an in. and in the case of a locomotive driver 68 in. in diam. the tire would be turned 68 times $\frac{1}{1000}$ or $\frac{68}{1000}$ of an in. smaller in diam, than the center on which it is to go. It is then heated so that it will expand sufficiently to slip on to the wheel before it cools. As it cools it contracts and grips the wheel. As it cannot return entirely to its original size this gripping power is sufficient to hold the tire in place during severe service. In case of a crank to be fitted on a 6-in, shaft the crank should be turned about 6_{1000} of an in. smaller than the shaft or else the shaft made \$\frac{6}{1000}\$ of an in, too large and the crank made just 6 in. in diam. This, as explained will make the proper allowance so that the crank will grip the shaft tightly after it has been shrunk on.

The amount of expansion of an object depends upon the kind of material, upon its length and upon the change in temperature of the object. Tables are prepared, which give us for different materials the amount of expansion per unit length, per degree change in temperature. These figures are called "coefficients of linear expansion."

The following figures for some of the most used materials are sufficiently accurate for our purposes.

COEFFICIENTS OF EXPANSION OF SOLIDS

Metal	Coefficient for 1° Fahr.
Aluminum (cast)	.00001234
Brass, cast	.00000957
Brass, plate	.00001052
Concrete: cement, mortar and pebbles	.00000795
Copper	.00000887
Iron, wrought	.00000648
Iron, cast	.00000556
Steel, cast	.00000636
Steel, tempered	.00000689

The coefficient for 1 Centigrade degree of temperature may be obtained from those above by multiplying them by $\frac{9}{5}$.

To calculate the amount of expansion the rule is, multiply the coefficient of expansion by the length of the object and by the number of degrees rise in temperature. If the length is taken in ft. the expansion will be in ft. If the length is in in. the answer will be in in., etc.

Example.—What is the amount of expansion of an 18-ft. brass rod when heated 150° Fahr.?

Solution.—Amount of expansion = Coefficient of expansion \times length of rod \times degrees rise in temperature.

Expansion = $.0000096 \times 18 \times 150$ or .0259 ft.

Example.—What will be the increase in diam. of a 64-in. diam. locomotive driver when heated 500° Fahr.? Solution.—The coefficient of expansion of steel is taken

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as .0000064. The expansion therefore equals .0000064 \times $64 \times 500 \text{ or } .2048 \text{ in.}$

Heat also causes liquids and gases to expand, but in the case of gases the change of volume depends upon the pressure of the gas as well as upon its temperature. Solids are liquefied by heat as in melting iron and steel and liquids are changed into vapors as in the production of steam in the locomotive boiler. 711

PROBLEMS

1. Find the expansion in an aluminum rod 20 ft. long if it is heated 750° Fahr.

2. What is the approximate expansion in the length of a steel bridge 2800 ft. long through a difference of temperature of 0° Fahr, in winter and 90° Fahr, in summer?

- 3. The walls of a building 40 ft. wide were drawn in and straightened by using iron rods through them, placing plates on the ends of the rods, heating the rods, tightening up the plates by means of nuts and allowing the rods to cool and contract, thus drawing the building together. If the rods each time changed in temperature 800° Fahr, how much was the building drawn together with each heating and tightening of the plates?
- 4. What is the amount of expansion in a 200-ft. span of copper wire which undergoes a change of temperature of 80° Fahr.?
- 5. An iron steam pipe changes in temperature from 180° Fahr. to 8° Fahr. What allowance must be made for the change in length if the pipe is 160 ft. long at 180° Fahr.?
- 6. If steel girders on a bridge are 60 ft. long at 20° Fahr., what is their length at 94° Fahr.
- 7. What allowance must be made for a crank to be fitted to a 5-in. diam. shaft?
- 8. Find the expansion in a 40-ft, rail between the temperature of - 20° Fahr, and 90° Fahr,
- 44. Specific Heat.—The specific heat of a substance means the number of heat units (B.t.u.) required to raise the temperature of any weight of the substance

1°, as compared with the number of heat units (B.t.u.) required to raise the temperature of the same weight of water 1°.

The following are examples of specific heat: Steam .48, iron .114, lead .0314.

This means that to raise the temperature of 1 lb. of iron 1° Fahr. requires only .114 B.t.u., whereas to raise the temperature of 1 lb. of water 1° Fahr. requires 1.0 (one) B.t.u.

45. Latent Heat.—Latent or "hidden heat" is that required to change the *condition* of a substance from a solid to a liquid or from a liquid to a vapor without changing its temperature.

To vaporize 1 lb. of water at 212° Fahr. to steam at 212° Fahr. at ordinary pressure requires 966 B.t.u. To melt 1 lb. of ice at 32° Fahr. to water at 32° Fahr. requires 142 B.t.u. of heat.

46. Mechanical Equivalent of Heat.—The mechanical equivalent of heat means the relation between units of work in foot-pounds, and equivalent heat units. We all know that friction produces heat and to overcome the friction work must be done. When 778 ft.-lb. of work are done, an amount of heat is produced equal to 1B.t.u. and this amount of heat is sufficient to raise the temperature of 1 lb. of water 1° Fahr.

Example.—How much heat is produced by 10,000 ft.-lb. of work?

Solution: $\frac{10,000}{778}$ or 12.8 B.t.u. Ans.

47. Heat Produced by the Electric Current.—The heat in B.t.u. produced by an electric current flowing through a resistance is equal to the square of the cur-

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rent in amperes times the resistance of the circuit in ohms times the time in sec. during which the current flows and times .00095 or

Heat in B.t.u. = (the current in amperes)² \times resistance in ohms \times seconds \times .00095

Example.—How much heat is produced by a current of 10 amperes flowing for 15 min. through a resistance of 22 ohms?

B.t.u. =
$$10 \times 10 \times 22 \times 15 \times 60 \times .00095$$
 or 1881.
Ans.

Working the other way round we can find the current required to produce a given amount of heat when flowing for a given time through a given resistance.

If H is the heat in B.t.u., I the current in amperes, R the resistance in ohms, and T the time in sec., the rule given above may be written,

$$H = I^2 \times R \times T \times .00095.$$

To find the current "I" in terms of the other quantities we have

$$I^2=rac{H}{R imes T imes .00095}$$
 and therefore $I=\sqrt{rac{H}{R imes T imes .00095}}$

 $VR \times T \times .00095$ Example.—What current in amperes is required to

produce 5000 B.t.u. in 10 min. when flowing through a resistance of 40 ohms?

Solution:

$$I = \sqrt{\frac{5000}{40 \times 10 \times 60 \times .00095}} \text{ or } \sqrt{\frac{5000}{24,000 \times .00095}}$$
This equals $\sqrt{\frac{5000}{22.8}}$ or 14.8 amperes.

To find "R" from the above rule in terms of the other quantities we have

$$R = \frac{H}{I^2 \times T \times .00095}$$

$$T = \frac{H}{I^2 \times R \times .00095}$$

Similarly

PROBLEMS

- 9. A Centigrade thermometer in a boiler room reads 40° what is the corresponding temperature Fahrenheit?
- 10. The difference between the temperature of two objects is 20 Fahr, degrees. Express this difference in Cent. degrees.
- 11. What effect would there be in the readings of a mercury thermometer if the tube were not of uniform bore?
- 12. A Fahrenheit thermometer gives the temperature of water as 95°. What is the corresponding temperature in degrees Cent.?
- 13. How much heat is required to raise 100 lb. of water 10° Fahr.?
- 14. If 10 cu. ft. of water change in temperature from 90° Fahr. to 75° Fahr., how many heat units are liberated?
- 15. It is desired to raise 5000 lb. of water from 60° Fahr. to 90° Fahr. and supposing 10 per cent. of the heat value of the coal to be used, how much coal is required if there are 12,000 B.t.u. per lb.?
- 16. How much heat is required to raise 100 lb. of iron from 70° Fahr. to 350° Fahr.? To how many foot-pounds of energy does this correspond?
- 17. Which requires more energy to raise the temperature of 10 lb. of iron from 0° to 100° Fahr. or to lift 5 tons of iron 1 ft. vertically?
- 18. If 1 lb. of coal contains 12,000 B.t.u. to what height could a 1200-ton weight be raised by utilizing all the heat value of the coal?
- 19. An engine raises a casting weighing 1½ tons through a vertical distance of 5 ft. What is the number of B.t.u. which this amount of work represents?

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- 20. How many B.t.u. are required to raise 50 lb. of lead from 72° Fahr. to 520° Fahr.?
- 21. A car weighing 15 tons is lifted by a crane through a vertical distance of 10 ft. Find the number of foot-pounds of work done and the number of B.t.u. which the work represents.
- 22. A certain coal gives 14,000 B.t.u. per lb. To what height would 5 lb. of this coal lift a weight of 8 tons if 40 per cent. of the B.t.u. were utilized?
- 23. If 2 tons of water are used per hr. in an ice plant in freezing water from an average of 70° Fahr., how many B.t.u. are extracted from this water per day of 10 hr.?
- 24. Which requires the more energy, to raise the temperature of 20 lb. of lead 100° Fahr. or to lift 4 tons of iron 12 ft. vertically?
- 25. If 1 lb. of coal contains 12,000 B.t.u. to what height could a 10-ton weight be lifted, if 5 per cent. of the heat value of the coal were utilized?
- 26. A wire cage weighing 1½ tons is lifted 900 ft. four times an hr. How many foot-pounds of work are done on the cage per hr.?
- 27. If 3 per cent. of the heat value of a 12,000 B.t.u. coal were utilized, how many pounds of coal would be used per hr. in lifting the cage in Prob. 26?
- 28. Find the heat produced by a current of 20 amperes flowing for 30 min. through a resistance of 24 ohms.
- 29. How long will it take a current of 10 amperes to produce 6000 B.t.u. when flowing through a resistance of 60 ohms?
- 30. What resistance is necessary in order to develop 4800 B.t.u. in 30 min. with a current of 8 amperes?

CHAPTER XVI

COMMON OR BRIGG'S LOGARITHMS

48. Definitions.—It is a good thing to become familiar with the use of logarithmic tables and common rules for using the logarithms of numbers.

In working out problems with logarithms we make use of a table of logarithms from which we can get the logarithm of any number of three, four or five figures, depending on how many figures the tables are made to give.

A logarithm of a number is expressed in two parts, one called the *characteristic* and the other the *mantissa*. The *characteristic* is obtained directly from the number and in the logarithm is the figure to the left of the decimal point. The *mantissa* is found in the tables and is the figure to the right of the decimal point in the logarithm.

49. The Characteristic.—When the number whose logarithm we are finding is greater than one the characteristic is positive, or plus. When the number is less than one the characteristic is negative, or minus. The characteristic is determined from the number of figures which make up the number whose logarithm we are finding, and is always one less than the total number of figures to the left of the decimal point. For instance, the characteristic of the logarithm for the number 3354 is 3 because there are four figures in the number. The characteristic of the logarithm for the number 384 is 2, for 24.43 it is 1, and for 7 is 0. There are all positive characteristics as the numbers were all greater than 1.

Table 1.—Common Logarithms

N.	0	1.	2	3	4	. 5	6	7	. 8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0756
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2381	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2712	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3114	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3097	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4109	4425	4440	4456
28	4472	4487	4502	4518	4533	4518	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871 5011 5145 5276 5403	4886	4900
31	4914	4928	4942	4955	4969	4983	4997		5024	5038
32	5051	5065	5079	5092	5105	5119	5132		5159	5172
33	5185	5198	5211	5224	5237	5250	5263		5289	5302
34	5315	5328	5340	5353	5366	5378	5391		5416	5428
35	5441	5453	5465	5178	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5617	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031 6138 6243 6345 6444	6042	6053	6064	6075	6085	6096	6107	6117
41	6128		6149	6160	6170	6180	6191	6201	6212	6222
42	6232		6253	6263	6274	6281	6294	6304	6314	6325
43	6335		6355	6365	6375	6385	6395	6105	6115	6425
44	6435		6454	6464	6474	6484	6493	6503	6513	6522
45 46 47 48 49	$\begin{array}{c} 6532 \\ 6628 \\ 6721 \\ 6812 \\ 6902 \end{array}$	$\begin{array}{c} 6542 \\ 6637 \\ 6730 \\ 6821 \\ 6911 \end{array}$	6551 6646 6739 6830 6920	6561 6656 6749 6839 6928	6571 6665 6758 6848 6937	6580 6675 6767 6857 6946	6590 6681 6776 6866 6955	6599 6693 6785 6875 6964	6609 6702 6791 6884 6972	6618 6712 6803 6893 6981
50	$\begin{array}{c} 6990 \\ 7076 \\ 7160 \\ 7243 \\ 7324 \end{array}$	6998	7007	7016	7024	7033	7042	7050	7059	7067
51		7084	7093	7101	7110	7118	7126	7135	7143	7152
52		7168	7177	7185	7193	7202	7210	7218	7226	7235
53		7251	7259	7267	7275	7281	7292	7300	7308	7316
54		7332	7340	7318	7356	7361	7372	7380	7388	7396
N.	0	1	2	3	4	Б	6	7	8	9

Table 1.—Common Logarithms.—Continued

N.	0	1	2	3	4		5	6	7	8	9
55	7404	7412	7419	7427	7435		7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513		7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589		7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664		7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738		7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810		7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882		7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952		7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021		8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089		8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156		8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222		8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287		8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351		8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414		8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476		8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537		8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597		8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657		8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716		8722	872 7	8733	8739	8745
75	8751	8756	8762	8768	8774		8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831		8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887		8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943		8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998		9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053		9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106		9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159		9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212		9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263		9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315		9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365		9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415		9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465		9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513		9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562		9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609		9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657		9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703		9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750		9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795		9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841		9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886		9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930		9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974		9978	9983	9987	9991	9996
N.	0	1	2	3	4	-	5	6	7	8	9

The negative characteristics for numbers less than one are determined in the same manner but they are expressed differently. Thus, the characteristic of the logarithm for the number .562 is -1 (minus one) and instead of writing -1 for the characteristic it is customary to change its form by adding 10, or a multiple of 10 to the characteristic, and then indicating the subtraction of the same number from this result. Thus a -1 characteristic is written 9. (mantissa) -10. For -10 + 9 = -1. The characteristic for the logarithm of the number .062 is -2 and is written 8. (mantissa) -10, since -10 + 9 = -1.

50. The Mantissa.—The mantissa consists of figures obtained from the tables and comprise portion of the logarithm to the right of the point or the decimal part of the logarithm and found by the use of the tables.

The total logarithm for a number is obtained by combining the mantissa read in the tables, with the proper characteristic.

51. Use of the Tables.—The tables give the mantissa of the number only, and are arranged to read the first two figures of the number of which we wish to find the logarithm under the column headed N. and the third figure of the number to the right of N. at the top of the table. For example, if we wish to find the logarithm of the number 215, look down the vertical column under N. for the first two figures of the number or 21 and then across the page horizontally to the vertical column headed with the third figure of the number or 5 and read the figures 3324 which is the mantissa of the logarithm for the number 215.

The characteristic for 215 is 2. Hence the logarithm of 215 is 2.3324.

The logarithm of the number 384 is 2.5843. (2 is the characteristic) (.5843 is the mantissa.)

The logarithm of the number .287 is -1.4579 (-1 is the characteristic) (.4579 is the mantissa).

The logarithm of the number 26 is 1.4150. (1 is the characteristic and .4150 is the mantissa.)

The logarithm of the number .236 is 9.3729 - 10. -1 or (9-10) is the characteristic and .3729 is the mantissa.

The logarithm of the number .0867 is 8.9380 - 10. -2 or (8 - 10) is the characteristic and .9380 is the mantissa.

The logarithm of 8.3 is 0.9191. 0 is the characteristic and 0.9191 is the mantissa.

52. To Find a Number Corresponding to a Logarithm.—We find the nearest logarithm to the one given and look across horizontally under column N. for the first two figures of the number and to the top of the column in which the logarithm occurs for the third figure. The characteristic will determine the number of figures to the left of the decimal point in the number. Thus: Find the number corresponding to the logarithm 2.3366. The nearest mantissa is .3365 occurring in the horizontal line opposite the number 21 in column headed N. and in the vertical column headed 7. Hence the number is 217. with the decimal point after the seven because the characteristic is 2 (the number of figures to the left of the decimal point in the number is one more than the characteristic).

There are four (4) general rules for the use of logarithms. These may be stated as follows:

Rule 1.—To multiply two numbers:

Add their logarithms and find the number in the table corresponding to the sum of their logarithms. Thus: Multiply 261 by 885

 $\log 261 = 2.4166$ $\log 885 = 2.9469$ Sum of $\log s = 5.3635$ Number = 231000.

Ans.

Pointing off one more figure in the number than the characteristic we have six figures before the decimal point in the answer.

PROBLEMS FOR PRACTICE

- 1. Multiply 235 by 360. 6. Multiply .279 by 56.3.
- 2. Multiply 898 by 210.
- 7. Multiply .076 by .005.
- 3. Multiply 635 by 359.
- 8. Multiply 9.15 by 12.4.
- 4. Multiply 110 by 236.
- 9. Multiply .028 by .144.
- **5.** Multiply 2.56 by 304.
- 10. Multiply 186 by .062.

Rule 2.—To divide two numbers:

Subtract the logarithm of the subtrahend from the logarithm of the minuend and find the number in the table corresponding to the difference of these logs. Thus:

Divide 836 by 2.10

 $\log 836 = 2.9222$

 $\log 2.10 = 0.3222$

Difference of logs = 2.6000 Number = 398. Ans.

Since the characteristic is 2.6000 we point off there figures in the number, the decimal point being placed after the eight.

EXAMPLES FOR PRACTICE

11. Divide 600 by 42.	16. Divide 1.04 by .02.
12. Divide 489 by 5.	17. Divide 98.1 by 12.5.
13. Divide 628 by 124.	18. Divide .126 by 6.26.
14. Divide 989 by 2.5.	19. Divide .166 by 1.25.
15. Divide 9.67 by .002	20. Divide 7.89 by 6.32.

53. To Raise a Number to a Power.—Multiply the logarithm of the number by the exponent or power to which the number is to be raised, and find the number in the table corresponding to the product thus obtained.

Example.—Find the value of (384)^{2.5}.

Solution:
$$\log 384 = 2.5843$$
.

 $2.5 \times 2.5843 = 6.4608$. The number corresponding to the logarithm 6.4608 = 2890000. Ans. The nearest number corresponding to the mantissa .4608 is 289. The characteristic 6 gives seven figures to the left of the decimal point.

Example.—Find the value of (.0325)3.

Solution: Log
$$.0325 = 8.5119 - 10$$

 $3 \times (8.5119 - 10) = 25.5357 - 30$
or -5.5357

From the tables the number nearest 5357 is 343. With a characteristic of -5 the answer is .0000343. Example.—Find the value of $(.305)^7$.

Solution: Log.
$$.305 = 9.4843 - 10$$

 $7 \times (9.4843 - 10) = 66.3901 - 70$
or -4.3901

From the tables the number nearest 3901 is 246. With a characteristic of -4 the answer is .000246.

PROBLEMS FOR PRACTICE

Find the value of

21. (628) ³	26. (76.3).02
22. (988)· ²	27. $(2.8)^{2.5}$
23. $(625)^{7.8}$	28. $(.002)^{2.18}$
24. (888) ^{1/8}	29. $(6.25)^{9.02}$
25. $(.867)^{2.62}$	30. $(0.0652)^{2.89}$

54. To Find a Root of a Number.

Divide the logarithm of the number by the index of root and find the number corresponding to the quont thus obtained.

 $\exists xample.$ —Find the value of $\sqrt[3]{286}$.

Solution:
$$\log 286 = 2.4564$$
 $\frac{2.4564}{3} = 0.8188$

From the tables the number nearest .8188 is 659.

With a characteristic of 0, the answer is 6.59 that is point off one figure in the number thus bringing the imal point after the six.

Zxample.—Find the value of $\sqrt[6]{.000625}$.

Solution: Log
$$.000625 = 6.7959 - 10$$

sefore dividing the logarithm by 6, we add and subet from the characteristic a number which is a multiple 10 and which will make the negative portion of the racteristic divisible by six without a remainder. add and subtract 50 the logarithm 6.7959 - 10 be-1es 56.7959 - 60.

'hen
$$\frac{56.7959 - 60}{6} = 9.46598 - 10$$
$$9.4660 - 10$$
$$-1.4660$$

rom the tables the number nearest 4660 is 292. 7ith a characteristic of -1 the answer is .292.

EXAMPLES FOR PRACTICE

31. $\sqrt{102}$	36. $\sqrt[6]{.260}$
32. $\sqrt[3]{3.85}$	37. $\sqrt[11]{4.26}$
33. $\sqrt[6]{360}$	38 . $\sqrt[9]{1.02}$
34. $\sqrt[7]{206}$	39. $\sqrt[3]{12.6}$
35. \(\frac{\cdot \delta}{\cdot \delta 265} \)	40 . $\sqrt[5]{32.6}$

MISCELLANEOUS PROBLEMS FOR PRACTICE

(To be solved by logarithms)

- **41.** Find the circumferences of circles whose diameters are as follows: 126 ft. 0 in. 7 ft. $6\frac{3}{4}$ in. 6 ft. 11.2 in. 1720 ft. 0 in. 7890 ft. 0 in. (Circumference = $3.14 \times D$ where D = diameter.)
- 42. What are the areas in sq. in. of circles with the following diameters: 62.2 in. 180 in. 9 ft. 6 in. 7 ft. $11\frac{5}{16}$ in. 5 ft. $10\frac{1}{2}$ in. (Area = $.785D^2$ where D = diameter.)
- 43. What are the diameters in in. of circles whose areas are: 6450 sq. in.? 5010 sq. in.? 4050 sq. in.? 7370 sq. in?

$$(D = \sqrt{\frac{\text{area}}{0.785}} \text{ or } D = \frac{1}{0.887} \sqrt{\text{area}} \text{ where } D = \text{diameter.})$$

44. Find the horsepower of a locomotive from the following rule:

Horsepower =
$$\frac{PLAN}{33,000} = \frac{130 \times 28 \times 24 \times 24 \times .785 \times 300 \times 2}{12 \times 33,000}$$

in which P=130 lb. per sq. in. pressure on the piston, cylinders 24 in. \times 28 in., N=300 strokes per min.

Work out the following problems by logarithms, showing all the work:

- **45.** $\frac{43.8 \times 2760 \times .97 \times 3.14}{7.85 \times 2.33 \times 42.6 \times .785}$
- **46.** $\sqrt{294}$. $\sqrt{4.84}$. $\sqrt[3]{3270}$. $\sqrt[3]{8.24}$.
- **47.** $(29.7)^2$. $(436)^3$. $(2.86)^2$. $(6.78)^3$.
- 48. Find the horsepower of a Pacific type locomotive with cylinders 24 in. \times 26 in., drivers 80 in., running at 40 m.p.h., and ef-



ctive steam pressure of 160 lb. per sq. in. using the following gures:

$$\begin{aligned} \text{Horsepower} &= \frac{PLAN}{33,000} \\ \underline{160 \times 26 \times 24 \times 24 \times .785 \times 40 \times 5280 \times 12 \times 2 \times 2} \\ \underline{12 \times 33,000 \times 80 \times 3.14 \times 60} \end{aligned}$$

- 49. An axle weighs originally 1050 lb. It loses 5% per cent. of its eight when turned down. Find the weight of the material recoved, the number of cubic inches of material removed, and the ral weight of the axle. The material weighs .28 lb. per cu. in.
- 50. Find the capacity in gal. and in cu. ft. of a cylindrical tank a ft. high and 10 ft., in diam.
- 51. Design a cylindrical tank to hold 20,000 gal.
- **52.** Design a coal bin to hold 300 tons of coal (one ton ocpies about 37 cu. ft.).
- 53. How many cylindrically shaped iron castings 4 in. diam. d 1½ ft. long can be made from 10 tons of metal. (Iron weighs 6 lb. per cu. in.)

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CHAPTER XVII

THE MEASUREMENT OF RIGHT TRIANGLES

55. Definition.—In the work which follows are rules for finding the sides and angles of right triangles, that is, those having a 90 degree (90°) or right angle. These rules are used to work out problems in the shop work as shown by examples which follow.

The rules for working out triangles (that is, figures with three angles and also three sides) form a branch of mathematics called "trigonometry," "tri" meaning

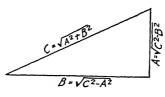


Fig. 104.

three, "gono" meaning side and "metry" measurement, or the measurement of three-sided figures. We need not think of the work necessarily as "trigonometry," but principally how it applies in working out problems that cannot be done by rules we have had before. We can use these rules also to do some of the problems quicker than by rules we have had up to this time.

56. Right Triangle Rule.—We have already had the rule that in a right triangle "the square of the hypothe-

nuse is equal to the sum of the squares of the other two sides," that is, referring to Fig. 104, if the hypothenuse is C, the height A, and the base B then

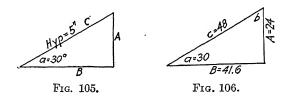
that is
$$(\text{hyp.})^2 = (\text{height})^2 + (\text{base})^2$$

$$C^2 = A^2 + B^2 \text{ from this}$$

$$C = \sqrt{A^2 + B^2}$$
also since
$$C^2 = A^2 + B^2$$

$$A^2 = C^2 - B^2$$
and
$$A = \sqrt{C^2 - B^2}$$
and also
$$B = \sqrt{C^2 - A^2}$$

That is when we have any two sides of a right triangle we can find the other side from one of these rules. It often is necessary, however, to find a side of a right triangle when we know only one other side and only one acute angle of the triangle. For example, in Fig. 105



the hypothenuse is 5 in. and the angle $a=30^{\circ}$. If we want to find either side A or B we cannot use the rules $A=\sqrt{C^2-B^2}$ and $B=\sqrt{C^2-A^2}$ since we know only one side in either of these equations. To find either side A or B we use certain ratios between the sides of the right triangle. These ratios have already been worked out for us in tables for different angles in the triangle.

If as in Fig. 106, angle $a = 30^{\circ}$. Side A = 24 and side C = 48.

$$\frac{\text{Side } A}{\text{Side } C} = \frac{\text{side opposite angle } a}{\text{hypothenuse}} = \frac{24}{48} = \frac{1}{2} = .5$$

57. Definition of Sine, Cosine, Tangent, and Cotangent.—This ratio of the opposite side and the hypothenuse is called the sine of angle a and for 30° it is .5. Similar ratios are worked out for all angles between 0° and 90° and put into tables for our use. If we divide the base B by the hypothenuse we have (adjacent meaning the side forming one side of the angle)

$$\frac{\text{side }B}{\text{side }C} = \frac{\text{side adjacent to angle }a}{\text{hypothenuse}} = \frac{41.6}{48} = .866.$$

This ratio is called the *cosine* of angle a. Similar ratios are worked out for all angles and put into tables for our use.

If we divide side A by side B we have

$$\frac{\text{side } A}{\text{side } B} = \frac{\text{side opposite angle } a}{\text{side adjacent to angle } a} = \frac{24}{41.6} = .577$$

This is called the *tangent* of angle a. Also

$$\frac{\text{side } B}{\text{side } A} = \frac{\text{side adjacent to angle } a}{\text{side opposite angle } a} = \frac{41.6}{24} = 1.73$$

This is called the *cotangent* of angle a.

These four ratios are what we will use in working out our triangles. They are called *functions* of the angle considered. Put in tabular form these functions are:

58. Tables of Rules for Functions of an Angle

$$\sin = \frac{\text{opposite side}}{\text{hyp.}} : \text{side opp.} = \sin \times \text{hyp.} : \text{hyp.} = \frac{\text{side opp.}}{\sin}$$

$$\cos = \frac{\text{adjacent side}}{\text{hyp.}} : \text{side adj.} = \cos \times \text{hyp.} : \text{hyp.} = \frac{\text{side adj.}}{\cos}$$

$$\tan = \frac{\text{opposite side}}{\text{adjacent side}} : \text{side opp.} = \tan \times \text{side adj.} : \text{side adj.} = \frac{\text{side opp.}}{\tan}$$

$$\cot = \frac{\text{adjacent side}}{\text{opposite side}} : \text{side adj.} = \cot \times \text{side opp.} : \text{side opp.} = \underbrace{\frac{\text{side adj.}}{\text{cot}}}$$

59. Calculation of Functions for 30° and 45°.—Referring to Fig. 107, in which angle $a=30^{\circ}$, angle $b=60^{\circ}$, side A=10, B=17.3, and C=20.

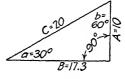
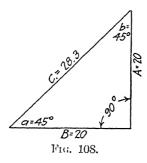


Fig. 107.

$$\sin a = \sin 30^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{10}{20} = .5$$
 $\cos a = \cos .30^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{17.3}{20} = .866$
 $\tan a = \tan 30^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{10}{17.3} = .577$
 $\cot a = \cot 30^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{17.3}{10} = 1.73$

the same way for Fig. 108, in which $a = 45^{\circ}$, b = A = 20, B = 20, and C = 28.3

$$\sin a = \sin 45^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{20}{28.3} = .707$$
 $\cos a = \cos 45^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{20}{28.3} = .707$
 $\tan a = \tan 45^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{20}{20} = 1.00$
 $\cot a = \cot 45^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{20}{20} = 1.00$



Explanation of Use of Tables.—These functions, ios of sides for a right triangle can be calculated for iven angle by constructing the right triangle and tring the length of its sides, but tables are prepared give us these ratios for all angles from 0° to 90° rate steps (see pages 133 to 156). These are the which we use in making calculations. In using tables the angle in degrees is read in the horizontal the top of the page for angles up to 44° and the es in the vertical columns to the left. Angles of over are read in the horizontal line at the bottom page for degrees and the minutes in the verti-

cal columns to the right. Thus page 134 gives by minute steps, the sines and cosines of all angles, from 1° to 4° inclusive and the sines and cosines of all angles from 85° to 88° inclusive. Page 135 gives by minute steps the sines and cosines of all angles from 5° to 8° inclusive and the sines cosines of all angles from 81° to 84° inclusive. Page 145 gives the tangents and cotangents of angles from 0° to 3° and the tangents and contangents of angles from 86° to 89° and page 146 gives the tangents and cotangents of angles from 4° to 7° and tangents and cotangents of angles from 82° to 85°. Referring to page 137.

The sine of $14^{\circ} 30' = .25038$ The sine of $13^{\circ} 20' = .23062$ The sine of $16^{\circ} 40' = .28680$

From page 143 the cosine of $50^{\circ} 20' = .63832$ (use the right-hand column of minutes and degrees at the *bottom* of the page).

The cosine of 73° 50' = .27843. The cosine of 0° 15' = .99999. The tables of tangents and cotangents, pages 145 to 146 are read in the same way.

PROBLEMS

- 1. From the tables of sines and cosines find the sines of the following angles: 15° , 30° , 60° , 90° (that is 89° 60'), 32° 10', 43° 42', 2° 4', 44° 50', 40° 55', 45° .
 - 2. Find the cosines of the same angles given in Prob. 1.
- 3. From the table of tangents and contangents find the *tangents* of the following angles: 14° 10′, 27° 57′, 42° 18′, 53° 12′, 63′, 28°, 45°, 30°, 17° 27′, 36°.
 - 4. Find the cotangents of the same angles given in Prob. 3.
- 5. Using the tables of functions complete the following table, finding first the angle from the function given, and then knowing the angle, find the other functions.

Prob. No.	Angle	Sine	Cosine	Tangent	Cotangent
1 2 3 4 5 6		.21644	.48226 .87462	6.31375	1.12369

61. Tables of Natural Sines, Cosines, Tangents, and Cotangents of Angles from 0° to 90° Varying by Minute Angles.

	1	00	1	11	1	00	1	11	1	0°	
_	SINE	COSINE		1	SINE	COSINE	1	1	SINE	COSINE	
0	.00000	I	60	21	.00011	.99008	39	41	0.7.7.10	1911 (Army	-
I	.00020	r	50	22		-99998	38		-01103	•99993	10
2	.00058	1	58	2.3		199998		42	-01222	•99993	18
3	.00087	I	57	2.1		-99998	37	43	.01251	-00002	17
4	.00116	I	50	25	.00727	-90007		44	•0128o	-00002	10
5	.00145	r	55	20	.00750	-90007	35	45	.01300	1,0000	15
6	.00175	I	5.1	27	.00785	-00007	34	40	85,510	100001	14
7	.0020.1	I	5.3	28	.0081.1	-90007	33	47	.01307	10000 r	1.3
8	.002,3,3	ı	52	20	.00844	00000	31		-01300	.00000	12
0	.00202	1	51	30	.00873	-00000	30	50	.01425	.00000	11
10	.00201	1	50	31	.00002			1	-01454	-99989	10
11	.00320	.00000	40	32		00000	20	51	-01483	-99989	Q
12	.00340	.00000	48	33	.00031 .00000	-00006	28	52	01513	.00080	8
13	.00378	.00000	-17	34	.00g8g	-00005	27	53	-01542	.00088	7
14	.00,107	.00000	16	35	81010	-90905	26	54	01571	-00088	6
15	.00.130	.00000	45	30	.010.17	-00005	25	55	*01(K)O	.00087	5
16	.00.1(15	-00000	1.1	37	.01076	300005	2.4	50	01030	-00087	4
17	.00.105	.00000	43	38	20110	400004	2.3	57	-01658	.00086	3
18	.00524	.00000	42	30	.01134	1,0000	22	58	·01687	-90986	2
19	.00553	.00008	11	40	.01104	4,0000 \$	21	50	-01716	-90085	1
30	.00582	800008	45	**	131114	.00003	50	00	.01745	-90085	o
7	No. Strategister, and		, ,,		! !						
	COSINE	Sinn	1	'	COMME	SINE	, 1	,			
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•	Sine	Cosine	SINE	Cosine	SINE	COSINE	SINE	Cosine	
0	.01745	.99985	.03490	-99939	.05234	.99863	.06976	99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861	.07005 .07034	-99754 -99752	59 58
2	.01803 .01832	.99984	.03548 .03577	.99937 .99936	.05292 .05321	.99858	.07063	·99750	50 57
3 4	.01862	.00083	.03606	99935	.05350	.99857	.07092	.99748	56
5	.01891	.99982	.03635	-99934	.05379	.99855	.07121	-99746	55
5	.01920	.99982	.03664	-99933	.05408	-99854	.07150	-99744	54
7 8	.01949	.99981	.03693	.99932 .99931	.05437 .05466	.99852 .99851	.07170	-99742 -99740	53 52
9	.01978 .02007	.99980	.03752	.99931	.05495	.99849	.07237	.99738	51
10	.02036	-99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.02065	-99979	.03810	.99927	.05553	.99846	.07295	•99734	49
12	.02094	.99978	.03839	.99926	.05582	.99844	.07324	-9973I	48
13	.02123	-99977	.03868	.99925	.05611 .05640	.99842	.07353	-99729 -99727	47 46
14	.02152 .02181	.99977 .99976	.03897 .03926	.99924	.05660	.99839	.07411	-99725	45
16	.02211	.99976	.03955	.99922	.05698	.99838	.07440	-99723	44
17	.02240	-99975	.03984	.99921	.05727	.99836	.07469	99721	43
	.02269	199974	.04013	.00010	.05756	.99834 .99833	.07498	.99719 .99716	42 41
19 20	.02298 .02327	-99974 -99973	.04042	.99918	.05785	.99831	.07556	99714	40
21	.02356	.99973	.04100	.99916	.05844	.99829	.07585	.00712	39
22	.02385	.99972	.04120	-99915	.05873	.99827	.07614	.99710	38
23	.02414	-9997I	.04150	-99913	.05902	.99826	.07643	.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705 .99703	36 35
25 26	.02472 .02501	.99969	.04217	.99911	.05960	.99821	.07730	.99703 .99701	34
27	.02530	.99968	.04275	.99900	.06018	.00810	.07759	•99699	33
28	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.02580	.99966	.04333	.99906	.06076	.99815	.07817	.99694 .99692	31 30
30	.02618	.99966	.04362	-99905	.06105			.99689	-
31 32	.02647 .02676	.99965 .99964	.04391	.99904	.06134	.99812	.07875	.99687	29 28
33	.02705	.00063	.04449	.99901	.06192	.00808	.07933	.99685	27
34	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25 24
36	.02792	.99961	.04536	.99897 .99896	.06279	.99801	.08040	.00676	23
37 38	.02850	-99959	.04504	.99394	.06337	-99799	.08078	.99673	22
39	.02879	-99959	.04623	.99893	.06366	-99797	.08107	.99671	21
40	.02908	.99958	.04653	-99892	.06395	-99795	.08136	.99668	20
41	.02938	-99957	04682	.99890	.06424	-99793	.08165	.99666 .99664	19
42 43	.02967	.99956 -99955	.04711	•99889 •99888	.c6453	.99792 .99790	.08194	.0066I	17
44	.03025	·99954	.04760	-99886	.06511	.99788	.08252	.99659	16
45	.03054	-99953	.04798	.99885	.06540	.99786	.08281	-99657	15
46	.03083	.99952	.04827	.99883	.06569	-99784	.08310	.00654	14
47 48	.03112	.99951	.04856 .04885	.99882 .99881	.06598	.99782	.08339 .08368	.99649	13
49	.03170	.99950	.04914	-99879	.06656	.99778	.08397	.99647	II
50	.03199	•99949	.04943	-99878	.06685	.99776	.08426	-99644	10
51	.03228	.99948	.04972	.99876	.06714	-99774	.08455	.99642	8
52	.03257	-99947	.05001	.99875	.06743	-99772	.08484	.99639	8
53 54	.03286	.99946 -99945	.05030	.99873	.06773	.99770	.08513	.99635	7
55	.03345	-99944	.05088	-99870	.06831	.99766	.08571	.99632	5 4
56	.03374	-99943	.05117	.99869	.06860	-99764	.08600	-99630	4
57 58	.03403	-99942	.05146	.99867	.06880	.99762	.08629	.99627	3 2
59	.03432	.9994I .99940	.05175	.99866	.06918	.99758	.08687	.99625	I
60	.03490	-99939	.05234	-99863	.06976	.99756	.08716	.99619	0
-	COSINE	SINE	Com	C	Com	Com	COSINE	SINE	—
·	COSINE	380 SINE	Cosine	Sine	Cosine	SINE	COSINE	5°	

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,	SINE	COSINE	SINE	COSINE	SINE	Cosine	SINE	Cosine	,
	.08716	.99619	.10453	-99452	.12187	-99255	.13017	.00027	60
0	.08745	.99617	.10482	-99449	.12216	.99251	.13946	.99023	59
2	.08774	.99614	.10511	.99446	.12245	.99248	-13975	.99019	58
3	.08803 .08831	.99612	.10540	-99443 -99440	.12274	.00244	.14004	.00015	57 56
4	.08860	.99607	.10507	•99437	.12331	.99237	.14001	.99006	55
5	.08880	.99604	.10626	-99434	.12300	.99233	.14000	.90002	54
7	.08918	.99602	.10055	-90431	.12389	.99230	.14110	98998 98994	53
8	.08947	-99599 -99596	.10684	.99.128 .99424	.12418	.99226	.14148	.08000	52 51
10	.00005	-99594	.10742	.99421	.12476	.99219	.14205	98986	50
11	-09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.00063	.99588	.10800	-99415	.12533	.99211	.14263	.08078	48
13	.00092	.99586	.10829	.99412	.12502	.99208	.14292	.08069	47
14	.00150	.00580	.10887	.99406	.12020	.00200	.14340	.98065	45
16	.00170	.99578	.1001c	.99402	.12649	.99197	.14378	9896r	44
17	.09208	-99575	.10045	-99399	.12678	.99193	.14407	.98957 .98953	43
18	.00237	.99572 .99570	.10073	-99396 -99393	.12700	.99186	.14464	.08048	41
20	-00205	.99507	.11031	.99390	.12764	.99182	.14493	.98944	40
21	.00324	-99564	.11000	.99386	.12793	.99178	.14522	.98040	39
22	.00353	.00562	.11080	-99383	.12822	.99175	.14551	-98936	38
23	.00382	-99559	.11118	.99377	.12851	.99171	.14580	.98931	37 36
24 25	.00411	-99550 -99553	.11176	99374	.12008	.99163	14637	-98923	35
26	1001100	-9955I	.11205	-99370	.12037	.00100	.14666	.98919	34
27	-00498	-99548	.11234	-90367	.12006	.90156	.14005	.98914	33
28 20	.00527	-995-15 -995-12	.11203	.90364	.12995	.99152	.14723	.08006	32
30	-09585	.995.40	.11320	-99357	.13053	-99144	.14781	.98002	30
31	.00614	-99537	.11340	-99354	.13081	11.100.	.14810	.98897	20
32	00042	-90534	.11378	-9035I	.13110	99137	.14838	-98893	28
33	.00700	-99531	.11.107	-00347 -00344	.13130	.00133	.14867 .14896	-08880 -08884	27
34 35	.00720	-99528 -99526	.11405	-90344	.13107	.00125	11025	.98880	25
36	.00758	-90523	-11404	99337	.13226	.00122	.14954	.08876	24
37	.00787	.99520	.11523	-90334	.13254	81100.	.14082	.08807	23
38 39	.00810	00517	.11552	-90331 -90327	.13283	41100	.150,10	.98863	21
40	.00874	.90511	.11000	.99324	.13341	.00100	.15000	.98858	20
41	.00003	.90508	.11638	.90320	.13370	.00102	.15007	.08854	ro
42	.000,32	.00500	.11007	-00317	.13300	80000	.15120	.08845	18
43	10000	.00503	.11725	.00314	.13.127	.00004 .00001	.15155	.98841	16
44 45	(1001.	-90407	11754	-00307	13485	.00087	.15212	.08830	1.5
46	100.18	1.01.00.	.11783	.00,303	-13514	.00083	.15241	-08832	1.4
47	.10077	101.00	.11812	.00,300	-13543	.00075	.15270	.08827 .08823	13
48 40	.10100	.00488	00811.	.00207	.13572	.00075	15327	.08818	11
50	.1010.4	.00482	.11898	.00200	.13020	.00007	.15350	.0881.4	10
51	.10102	.00.170	.11927	.00286	.13658	.00063	.15385	.g88cg	8
5.2	10331	.00.170	07011.	.00283	-13087	-00250	.15414	.08805 .08800	7
53 54	.10250	-00473	.11085	.00270	.13716	-00055 -00051	-15442	.08700	6
55	.10308	.00.170	12013	.00272	-13773	-000.17	15500	.08701	5
50	.10337	4.00,46.4	.12071	.00200	.13852	-00043	.15520	.08787	4
57 58	.10300	101.00.	.12100	.00265	1,88,1.	.00030 .00035	.15557	.08782	3 2
50 50	.10305	-00458	.12120	.00258	13880	-00035	.15500	08773	1
tio	.10.153	-90452	.12187	.90255	.13017	.00027	.15043	.08700	0
-	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	,
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•	SINE	Cosine	SINE	Cosine	SINE	COSINE	SINE	Cosne	
0	.15643	.98769	.17365	.98481	.19081	.98163 .98157	.20791	.97815	60
I	.15672	.98764 .98760	.17393 .17422	.98476 .98471	.19138	.08152	.20848	.97803	59 58
3	.15701 .15730	.98755	.17451	.98466	.19167	.98146	.20877	-97797	57 56
4	.15758	.98751	.17479	.98461	.19195	.98140	.20905	-9779I	56
5	.15787	.98746	.17508	.98455 .98450	.19224	.98135 .98129	.20933	.97784	55 54
	.15816	.98741 .98737	.17537	.98450	.19252	.98124	.20000	-97772	53
7 8	.15873	.98732	.17594	.98440	.19309	.98118	.21019	.97766	52
9	.15902	.98728	.17623	.98435	.19338	.98112	.21047	.97760	51
10	·15931	.98723	.17651	.98430	.19366	.98107		-97754	50
II	.15959	.98718	.17680	.98425	.19395 .19423	.98101 30080.	.21104	.97748 .97742	49 48
12	.15988	.98714	.17708	.98420	.19423	.98090	.21161	.97735	47
14	.16046	.98704	.17766	.98409	.19481	.98084	.21180	-97729	46
15	.16074	.98700	.17794	.98404	.19509	.98079	.21218	-97723	45
16	.16103	.98695 .98690	.17823	-98399	.19538	.98073 .98067	.21240	.97717 .07711	44
17 18	.16132	.98689	.17852	.98394 .98389	.19595	.98061	.21303	-97705	42
10	.16180	.08681	.17909	.98383	.19623	.98056	.21331	.97698	4I
20	.16218	.98676	·17937	.98378	.19652	.98050	-21360	.97692	40
21	.16246	.98671	.17966	.98373 .98368	.19680	.98044	.21388	.97686	39
22	.16275	.98667	·17995	.98368	.19709	.98039	.21417	.97680 .97673	38
23 24	.16304 .16333	.98662 .98657	.18023	.98362 .98357	.19737 .19766	.98027	.21474	.97667	37 36
	.16361	.98652	.18081	.08352	.19794	.98021	.21502	.97661	35
25 26	•16390	.98648	.18100	.98347	.19823	.98016	.21530	-97655	34
27 28	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
20	.16447 .16476	.98638 .98633	.18100	.98336 .98331	.19308	.97987	.21616	.97636	3r
30	.16505	.98629	.18224	.98325	.19937	.97992	.21644	.97630	30
31	.16533	.98624	.18252	.98320	.19965	-97987	.21672	.97623	20
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617 .97611	28
33	.16591 .16620	.08614	.18300 .18338	.98310	.20022 .20031	.97975 .97969	.21729	.97604	27 26
34 35	.16648	.98604	.18367	.98299	.20070	.97963	.21786	.97598	25
36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	-97592	24
37 38	.16706	.98595	.18424	.98288	.20136	-97952	.21843	.97585	23
38 39	.16734 .16763	.98590 .98585	.18452	.98283	.20165	.97946	.21899	-97579 -97573	21
40	.16792	.98580	.18500	.98272	.20222	-97934	.21928	.97566	20
41	.16820	.98575	.18538	.98267	.20250	.97928	.21956	.97560	29
42	.16849	.98570	.18567	.98261	.20279	.97922	.21985	-97553	18
43	.16878	.98565 .98561	.18595	.98256 .98250	.20307	.97916	.22013	-97547 -97541	17
44 45	.16935	.98556	.18652	.08245	.20330	-97905	.22070	-97534	15
46	.16964	.98551	.18681	.98240	.20393	.97899	.22008	.97528	14
47 48	.16992	98546	.18710	.98234	.20421	-97893	.22126	-97521	13
48 49	.17021	.98541 .98536	.18738	.98229	.20450	.97887	.22155	-97515 -97508	12
50	.17078	.98531	.18707	.98218	.20507	.97875	.22212	.97502	10
51	.17107	.98526	.18824	.08212	.20535	.97869	.22240	-97496	9
52	.17136	.98521	.18852	.98207	.20563	.97863	.22268	-97489	- 8
53	.17164	.98516	.18881	.98201	.20592	.97857	.22297	-97483	7
54 55	.17193	.98511	.18910	.98196	.20020 .20040	.97851 -97845	.22325	.97476 .97470	5
56	.17250	.98501	.18967	.98185	.20677	.97839	.22382	.97463	4
57	.17279	.98496	.18995	.98179	-20706	.97833	.22410	-97457	3
58	.17308	.98491 .98486	.19024	.98174	.20734	.97827	.22438	-97450	2 I
59 60	.17365	.98481	.19052	.08163	.20703	.97815	.22407	.97444 .9743 7	a
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		07	437	.24	192		7030		882	.00	5593 5585		592	.00	8116	59	
0	.22495	.97	430		220	-97	7023	-25	5938	.00	6578		620	.96	110	58 57	
τ 2	22552		424		249	.9,	7008	.2	5966		6570		648		0004	56	
3	.22580		7417		1305	-9	7001	.2	5994		6562 6555		7704	.90	5086	55	
4	.22608 .22637		7404	.24	1333	-0	5994		6022 ! 6050		6547		7731		5078	54	
5	.22665	.97	7398	.2.	4362	-9	6987 6980	.2	6079	و. ا	6540	.2	7759		5070 6062	53 52	
	.22693	-91	7391	.2.	4390 4418	٠.	6973	.2	6107		6532		7787 7815		6054	51	
7 8	.22722	1.9	73 ⁸ 4 7378	.2	4446		6966		6135 6163		6517	.2	7843		6046	50	
10	.22778	1.9	7371	.2	4474		6959		6101		6509	.2	7871	.9	6037	49	
11	.22807	1.9	7365		4503		6952		6219	9.	6502		7899		6029 6021	48 47	
12	.22835	.9	7358		4531 4559		6937	.2	6247	٠.	6494		7927 7955		6013	46	
13	.22863		7351	.2	4587		6930		26275	1 .9	96486 96479	.2	7983		6005	45	
14	.22892	0.9	7338	.2	4615	٠.	6923		26303 2633I		9647I	.2	8011		5997	44	
15 16	.22948	1 .9	7331		4644 4672		6909		26359		96463		8039 8067	1 .9	95989 95981	43	
17	.22977	1.9	7325		24700	1 .	06002		26387	1 .	96456 96448		28005		05972	41	
18	.23005		7311		24728	1	96894		26415 26443	1:	96440		28123		5964	40	
20	.23062	1.9	7304		24756		96887	u i	2647I	1	06433		28150		95956	39	3
21	.23090		97298		24784	1.	96880 96873		26500	١.	96425		28178		95948	34	7
22	.23118		97291		24813 24841	1:	96866	И.	26528		.96417		28206 28234		95940 95931	3	6
23	.23146		97284 97278		24869	١.	96858	-	26556	ı	.96410 .96402		28262		95923	3	5
24	.23175		0727I	١.	24897		96851 96844		.26584 .26612		.96394		28290		95915	3	
25 26	.23231		97264		24925 24954		.96837	₩.	26640		.96386		.28318 .28346		95997 95898	3	2
27 28	.23200		97257 97251		24982	1.	.96829		.26668		.96379 .96371		28374	. .	95890	3	I
28	.23288	5 .	97244	11 .	.25010		.96822		.26696 .26724		.96363		28402	: -	.95882	1 -	0
29 30	.2334		97237		.25038	1	.96815	ш	.26752		.06355		.28429		95874		8
31	.2337		.97230	1	.25066 .25094	1	.06800	11	.26780)	.96347		.28457 .28485		.95865 .95857		7
32	.2340		.97223	11	.25122		.06793	11	.26808		.96340	- 11	.28513	2	.05840		26
33			.07210	11	.25151		.96786	11	.2686	2	.96324		.2854	r	.95841	.] 2	25
34 35	.2348	6	.97203	11	.25179		.96778	11	.2689	2	.96316	5	.2856	2	.95832		24 23
36	.2351	4	.97196	11	.25235		.96764	. 11	.2692		.96308	-	.2859	5	.95816	• :	22
37	7 -2354		.97182	11	.25263	3	.96756	1	.2694		.96293		.2865	2	.95807		21
38	-2359	9	.07176	- 11	.2529		.96749		.2700		.9628		.2868		.95799	1	20
49	.2362	27	.97169		-25320		.96734	- 1)	.2703	2	.9627	7	.2870		•95793 •9578	- 1	19 18
4			.97162	:	.2534	6	.96727		.2700	io	.9626	2	.2873	4	·9577	- 1	17
4:			.97148	3	.2540	4	.96719	2	.2708	6	.9625	3	.2879	2	.9576	6	16
4.			.97141	:	-2543	2	.9671	2	.2714		.9624	6	.2882		-9575		15 14
4	5 -237		.97134		.2546	8	.9669		.271	72	.9623	8	.288		.9574 .9574		13
4	6 .237	97	.97120	ś	.2551	6	.9669		.272	28	.9623	2	.280	03	-9573	2	12
4	8 1 .238	53	.9711	3	.2554		.9668		.272	56	.0621	4	.280		-9572		11
4	19 .230	82	.97100	0	.2557	3	.0666		.272	84	.9620		.289		·957 I	1	9
_	-239			- 1	.2562		.0666	0	.273		.9619	98	.289		.9579	8	8
	239 52 -239)38 566	.9709 .9708	8	.256	57	.9665		.273	40	.961	82	.200		.0500	00 1	7 6
	52 -239 53 -239		.9707	9	.256	85	.9662	15	.273		.961		.290	70	.9568	31	5
	54 .249	023	.9707	2	.257 .257		.9663	30	.274	124	.961		.290	ეგ ენ	.956	54	4
		051 070	.9700	8	-257	69	.966:	23	.274	152	.961		.201		.956	56	3
	57 .24	108	.9705	ξī	.257	იგ	.966	15	.274	508	.961		.201	82	.956	47	2 1
	58 -24	136	.9704	14	.258	20	.966		.27	536	.061	34	.20		.956 .956	30	o
	59 .24	104	.970	37 30	.258	82	.905		.27	564	.901	20	.29:	-31	-		-
					-11		SIN	117	Cos	INE	Siz	1E	Cos	INE	SIN	E	\
	' Cos	SINE	Sin	E	Cos	INE	750		11 200		74°		IJ	7	'3°		•
	i	7	'6°		11	•											

1	1			3°		9° 1	-	0°	١.
	SINE	Cosine	SINE	Cosine	SINE	Cosine	SINE	COSINE	
0 I 2	.29237 .29265 .29293	.95630 .95622 .95613	.30902 .30929 .30957	.95106 .95097 .95088	-32557 -32584 -32612	.94552 .94542 .94533	.34202 .34229 .34257 .34284	.93969 .93959 .93949 .93939	50 58 58
3 4 5 6	.29321 .29348 .29376 .29404	.95605 .95596 .95588 .95579	.30985 .31012 .31040 .31068	.95079 .95070 .95061 .95052	.32639 .32667 .32694 .32722	-94523 -94514 -94504 -94495	.34311 -34339 .34366 -34393	.93939 .93929 .93919 .93909 .93899	56 55 54 53
7 8 9 10	.29432 .29460 .29487 .29515	.95571 .95562 .95554 .95545	.31095 .31123 .31151 .31178	.95043 .95033 .95024 .95015	.32749 .32777 .32804 .32832	.94485 .94476 .94466 .94457	.34421 .34448 .34475	.93889 .93879 .93869	52 51 50
11 12 13 14 15 16	.29543 .29571 .29599 .29626 .29654 .29682	.95536 .95528 .95519 .95511 .95502 .95493	.31206 .31233 .31261 .31289 .31316 .31344 .31372	.95006 .94997 .94988 .94979 .94970 .94961	.32859 .32887 .32914 .32942 .32969 .32997 .33024	.94447 .94438 .94428 .94418 .94409 .94399	.34503 .34530 .34557 .34584 .34612 .34639 .34666	.93859 .93849 .93839 .93829 .93819 .93809	49 48 47 46 45 44 43
18 19 20	•29737 •29765 •29793	.95476 .95467 .95459	.31399 .31427 .31454	•94943 •94933 •94924	•33051 •33079 •33106	.94380 .94370 .94361	.34694 .34721 .34748	.93789 .93779 .93769	42 41 40
21 22 23 24 25 26	.29821 .29849 .29876 .29904 .29932 .29960	.95450 .95441 .95433 .95424 .95415 .95407	.31482 .31510 .31537 .31565 .31593 .31620	.94915 .94906 .94897 .94888 .94878 .94869	-33134 -33161 -33189 -33216 -33244 -33271	.94351 .94342 .94332 .94322 .94313 .94303	•34775 •34803 •34830 •34857 •34884 •34912	93759 93748 93738 93728 93718	39 38 37 36 35 34
27 28 29 30 31	.29987 .30015 .30043 .30071 .30098	.95398 .95389 .95380 .95372 .95363	.31648 .31675 .31703 .31730 .31758	•94860 •94851 •94842 •94832 •94823	•33298 •33326 •33353 •33381 •33408	.94293 .94284 .94274 .94264	.34939 .34966 .34993 .35021	.93698 .93688 .93677 .93667	33 32 31 30
32 33 34 35 36 37 38 39	.30126 .30154 .30182 .30209 .30237 .30265 .30292 .30320 .30348	•95354 •95345 •95337 •95328 •95319 •95310 •95301 •95293 •95284	.31786 .31813 .31841 .31868 .31896 .31923 .31951 .31979 .32006	.94814 .94805 .94795 .94786 .94777 .94768 .94758 .94749	.33436 .33463 .33490 .33518 .33545 .33573 .33000 .33627 .33655	.94245 .94235 .94225 .94215 .94206 .94196 .94186 .94176	.35075 .35102 .35130 .35157 .35184 .35211 .35230 .35266 .35293	.93647 .93637 .93626 .93616 .93606 .93596 .93585 .93575	28 27 26 25 24 23 22 21 20
41 42 43 44 45 46 47 48	.30376 .30403 .30431 .30459 .30486 .30514 .30542	.95275 .95266 .95257 .95248 .95240 .95231 .95222	.32034 .32061 .32089 .32116 .32144 .32171 .32199 .32227	.94730 .94721 .94712 .94702 .94693 .94684 .94674	.33682 .33710 .33737 .33764 .33792 .33819 .33846	.94157 .94147 .94137 .94127 .94118 .94108 .94098 .94088	•35320 •35347 •35375 •35402 •35429 •35456 •35484 •35511	.93555 .93544 .93534 .93524 .93514 .93503 .93493 .93483	19 18 17 16 15 14 13
49 50 51 52	•30597 •30625 •30653 •30680	.95204 .95195 .95186	.32254 .32282 .32309	.94656 .94646 .94637	.33901 .33929 .33956	.94078 .94068 .94058	.35538 .35565 .35592	.93472 .93462 .93452	11 10 9 8
53 54 55 56	.30708 .30736 .30763 .30791	.95177 .95168 .95159 .95150 .95142	.32337 .32364 .32392 .32419 .32447	.94627 .94618 .94609 .94599	•33983 •34011 •34038 •34065 •34093	.94049 .94039 .94029 .94019	.35619 .35647 .35674 .35701 .35728	.93441 .93431 .93420 .93410 .93400	7 6 5 4
57 :8 59 60	•30819 •30846 •30874 •30902	.95133 .95124 .95115 .95106	.32447 .32474 .32502 .32529 .32557	.94580 .94571 .94561 .94552	.34120 .34147 .34175 .34202	.93999 '93989 -93979 -93969	-35755 -35782 -35810 -35837	-93389 -93379 -93368 -93358	3 2 1
-,-	Cosine 7	SINE 2°	Cosine 7	SINE 1°	Cosine 7	SINE	Cosine 69	SINE	•

	21	0	22	° 1	23	3°	24	lo I	
	SINE	Cosine	SINE	Cosine	SINE	COSINE	SINE	Cosine	_
0	.35837	.03358	-3746t	.02718	.39073	.02050	.40674	-91355	60
ı	.35864	.93348	.37488	.92707	.39100	.92039 .92028	.40700 .40727	.91343 .91331	59 58
2	.35891	.93337 .93327	.37515 .37542	.02086	.39153	.02016	.40753	.91319	57
3	.35918	.93316	-37509	.92675	.39180	.02005	-40780	.91307	56
4	.35945 .35973	.93306	-37595	.92664	.39207	.91904	.40806	.91205	55
5	.36000	93295	.37022	.92653	-39234	-91982	-40833	.91283	54
	.36027	.93285	37649	.92642	.39200	-91971	.40860 .40886	-91272	53
7 8	.36054	.93274	.37676 .37793	.92631	.39287	.91959 .91948	.40000	.91260	52 51
9	.36081	.93264 .93253	.37730	.02000	.39341	.91936	.40030	.01236	50
10	.36108	.03243	·37757	.92598	30367	.91925	.40966	.91224	49
11	.36135	.03232	37784	.92587	39394	.01014	.40002	.91212	48
12	.36102	.93222	.37811	.02576	-39421	-01002	.41019	.91200	47
13 14	.36217	.93211	.37838	.02565	-39448	.91891	.41045	.91188	46
15	.36244	.03201	-37865	.92554	-39474	-91879	.41072	.91176	45
ıŏ	.36271	.03100	.37892	.92543 .92532	.30501 -30528	.91868 .91856	.41098 .41125	.91164 .91152	44
17	.36298	.03180	.37010 .37040	.02521	-39555	.918.45	-41151	.01140	43
18	.36325	.93159	-37973	.02510	-39581	.91833	.41178	.91128	41
19 20	.36352	.93148	-37999	.92499	.39008	.91822	-41204	.91116	40
21	.36406	-03137	.38026	.02488	-39635	.01810	.41231	.91104	39
22	.36434	.03127	.38053	.02477	.3960x	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.02400	.39688	.91787	-41284	.91080	37
24	.36488	.03100	.38107	-92455	-39715	.91775	-41310	.91068	36
25	.36515	.03005	.38134	.02444	-39741 -39768	.91764 .91752	-41337 -41363	.91044	35 34
26	.36542	.03084	.38188	.02421	39705	.01741	.41300	.01032	33
27 28	.36569 .36596	.030/4	.38215	.02410	.30822	.01720	-41416	.01020	32
20	.36623	.03052	.38241	.02300	.308.48	.91718	-41443	.01008	31
30	.36650	.03042	.38268	.92388	-39875	.91700	-41469	.90996	30
31	.36677	.0,30,31	.38205	.92377	.30002	.0160.1	.41496	.90984	20
32	.36704	.0,3020	.38322	.02300	-39928	.01683	.41522	.90972	28
33	.30731	.0,3010	38349	-02355 -02343	-39955 -39982	.01671	-41549 -41575	.90948	27
3.1	.36758	.02088	.38376	.02332	10008	.016.48	.41002	.00036	25
35 36	.36785	.02078	.38430	.02321	.40035	.01636	41628	.90924	24
37	.36839	.02007	38450	.02310	.40002	.01625	-41655	11000.	23
38	.36867	.02050	.38483	.02200	10088	.91613	-41681	.90899	22
39	.36804	.02045	.38510	.02287	10115	.01601	.41707	.00875	20
40	.36921	.92935	-38537	02270	11.101.		.41760	.00863	10
41	.36048	.02024	.38564 .38591	.02265	.40105	.01578	-41787	.90851	18
42	.36975	.02013	38617	.02254	.40221	91555	.41813	.90839	17
43 44	.37002	.02802	.38644	.02231	.402.48	.01543	.41840	.00820	16
45	.37056	.02881	.38071	.02220	.40275	.01531	.41866	.00814	15
46	37083	.02870	.38098	.02200	.,10,301	.01510	.41802	.00802	1.1
47	.37110	.02850	38725	.02180	.40328	.01508	-41010 -41045	.00778	12
48	-37137	.02840	.38752 .38778	.02175	-40355 -40381	CA 48.1	-41072	.00706	11
49 50	.3716.1	.02827	38805	.02104	.40408	.00 472	11998	.99753	10
		.02816	.38833	.02152	-10434	.01461	.42024	.007.11	8
5 I 52	.37218	.0.805	38850	.02141	10101	.01440	;2051	.00720	
53	.37272	.02704	.38886	02130	.40.488	.01437	-42077	.00717	7 6
54	.37290	.02784	.38012	.02110	.40514	.01425	.42101	.00002	5
5.5	.37326	-02773	-38030	.02107	10541	01103	.42130 .42150	.00002	3
56	-37353	.02762	.38000	.02000	40504	.01.102	42183	81000	3
57	.37380	.0.2751	.38003 .30020	.0.3073	12007	-01378	.42200	.00655	2
58 59	·37407 ·37434	.02720	300.40	.02002	100.17	.01300	42235	.00043	1
(io	37461	.02718	.30073	.0.:050	.40074	.01355	-12202	.00031	
_			43	(******	C	SINE	COSINE	SINE	1
′	COSINE	SINE	COSINE	7°	Cosine	66° SINE	COSINE	550	1
	ı G	80 5	u u	11	11 C	,,,		, ,	

	2	5°	11 2	6°	1 2	7°		8°	l
,	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	
0	-42262	.90631	.43837	.89879	-45399	.89101	-46947	.88295	60
I	-42288	.90618	.43863	.89867	-45425	.89087 .89074	.46973 .46999	.88267	59 58
2	-42315	.90606	-43889	.89854	-4545I	.80061	.47024	.88254	20
3	-4234I	-90594	-43916	.89841 .80828	•45477 •45503	.89048	.47050	.88240	57 59
4	-42367	.90582	-43942 -43968	.80816	-45529	.89035	.47076	.88226	55
5	.42394 .42420	.90569	·43994	80803	45554	.89021	.47101	.88213	54
	.42446	-90557 -90545	.44020	.89790	.45580	.89008	-47127	.88199	53
8	-42473	.90532	.44046	.80777	.45606	.88995	-47153	.88185	52
9	-42499	.90520	.44072	.89764	.45632	.8898r	47178	.88172	51
10	.42525	.90507	.44098	.89752	-45658	-88968	.47204	.88158	50
11	-42552	-90495	.44124	.80739	·45684	-88955	-47229	.88144	49
22	-42578	-90483	-4415I	.89726	-45710	-88942	·47255	.88130	48
13	.42604	-90470	-44177	.89713	-45736	.88928 .88915	-47281	.88117 .88103	47 46
14	.42631	.90458	-44203	.89700	-45762	.88902	-47306	.88080	45
15	.42657	.90446	-44229	.89687 .89674	-45787 -45813	.88888	·47332 ·47358	.88075	45
16	-42683 -42700	.90433 .90421	-44255 -44281	.89662	45820	.88875	.47383	.88062	43
17	.42736	.90421	. 44307	.89649	.45839 .45865	.88862	.47409	.88048	42
IO	-42762	190396	•44333	.80636	.45801	.88848	•47434	.88034	41
20	-42788	.90383	•44359	.89623	-45917	.88835	.47460	.88020	40
21	-42815	.90371	-44385	.89610	.45942	.88822	.47486	.88006	39
22	-42841	.90358	.44411	.89597	.45968	.88868	-475II	.87993	38
23	-42867	.90346	•44437	.89584	·45994	.88795	•47537	.87979	37
24	-42894	•90334	-44464	.8957I	.46020	.88782 .88768	47562	.87965 .87951	36
25	-42920	.9032I	-44490	.89558	.46046 .46072	.88755	.47588	87937	35 34
26	-42946	-90300	-44516	.89545 .89532	.46097	.88741	.47639	.87923	33
27 28	.42972 -42999	.90296 .90284	.44542 .44568	.89519	.46123	88728	.47665	.87909	32
20	.43025	.90271	44594	.89506	.46149	.88715	.47690	.87896	3 I
30	-4305I	90259	.44620	,89493	.46175	.88701	.47716	.87882	30
31	-43077	.90246	.44646	.89480	46201	.88688	-4774I	.87868	20
32	-43104	.90233	.44672	.89467	.46226	.88674	-47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661 .88647	.47793 .47818	.87840 .87826	27.
34	.43156	-90208	-44724	.89441 .89428	.46278 .46304	.88634	.47844	.87812	25
35 36	.43182 .43200	.90196	-44750 -44776	.80415	.46330	.88620	.47869	.87798	24
37	·43235	.90171	.44802	.80402	.46355	.88607	.47805	87784	23
37 38	-43261	.90158	.44828	.89389	.46381	.88593	-47920	.87770	22
39	.43287	.90146	.44854	89376	.46407	.8858o	.47946	.87756	21
40	·43313	.90133	.44880	.89363	.46433	.88566	·4797I	.87743	20
41	-43340	.90120	.44906	.89350	.46458	.88553	-47997	.87729	10
42	.43366	.90108	+44932	·89337	46484	.88539	.48022	.87715	18
43	·43392	.90005	-44958	.89324	.46510	.88526	.48048	.87701	17
44	-43418	.90082	-44984	.89311	.46536	.88512	.48073	.87687	15
45 46	·43445 ·4347I	-90070 -90057	.45010 .45036	.89298 .89285	-46561 -46587	.88499 .88485	.48099 .48124	.87673 .87659	14
17	·43471 ·43497	.90037	.45062	.89272	.46613	.88472	.48150	.87645	13
47 18	·43523	-90032	.45088	80250	.46639	.88458	.48175	.87631	12
49	-43549	-90019	-45114	.89245	46664	.88445	.48201	.87617	II
50	·43575	.90007	-45140	.89232	.46690	.88431	-48226	.87603	10
5 I	-43602	.89994	.45166	89219	.46716	.88417	.48252	.87589	8
52	-43628	89981	45192	89206	-46742	.88404	.48277	.87575	8
53 54	.43654 .43680	.89968 .89956	.45218	.89193 .89180	-46767	.88350	.48303	.87561	7
55 55	·43706	.89930	.45243 .45269	.80167	.46793 .46819	.88377 .88363	.48328 .48354	.87546 .87532	5
56	·43733	.89930	·45205	.89153	.46844	.88349	-48379	.87518	4
57 58	-43759	.89918	45321	.89140	.46870	.88336	.48405	.87504	3
58	-43785	89905	-45347	.89127	-46896	.88322	.48430	.87490	2
59	-43811	-89892	-45373	89114	-4692I	-88308	-48456	.87476	1
60	-43837	.89870	-45399	.89101	.46947	.88295	.48481	.87462	٥
′	COSINE	SINE 4º	COSINE	SINE	Cosine 62	SINE	Cosine 6	SINE	,
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•	Sine	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	<u>'</u>
0	-48481	.87462	.50000	.86603	.51504	.85717	-52992	.84805	60
1	. 48506	.87448	.50025	.86588	-51529	.85702	-53017	84789	59
2	-48532	87434	.50050	86573	-51554	.85687	-53041	.84774	58
3 4	-48557	87420	.50076	.86559	-51579	.85672	.53066	84759	57 56
4	·48583	.87406	.50101	.86544	.51604	.85657	-53091	.84743 .84728	55
5	-48608	.87391	.50126	86530	.51628	.85642 .85627	·53115 ·53140	.84712	55 54
0	-48634	.87377	.50151	.86515	.51653	.85612	.53164	.84607	53
7 8	48659	.87363	.50176	.86501 .86486	.51703	.85597	.53189	.84681	52
9	48684	.87349	.50201	.86471	.51728	85582	-53214	84666	51
10	.48710 .48735	.87335	50227 50252	86457	.51753	.85567	.53238	.84650	50
		.87321	1		.51778	.85551	.53263	.84635	49
11	·48761	.87306	.50277	.86442 .86427	.51803	.85536	.53288	.84619	48
13	.48786 .48811	.87292 .87278	.50302	.86413	.51828	.85521	.53312	.84604	47
14	48837	.87276	.50327	.86398	.51852	.85506	-53337	.84588	46
15	48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	48888	.87235	.50403	.86369	-51902	-85476	.53386	.84557	44
17	.48013	.8722I	.50428	.86354	.51927	85461	-53411	.84542	43
18	·48938	.87207		.86340	.51952	.85446	-53435	.84526	42
19	.48964	.87193	.50453 .50478	86325	-51977	-8543I	-53460	.84511	41
20	48989	87178	.50503	.86310	.52002	.85416	• 53484	.84495	40
21	·49014	.87164	.50528	.86295	.52026	.85401	-53509	.84480	39
22	-49040	87150	-50553	.86281	.52051	.85385	·53534	.84464	38
23	-49065	.87136	.50578	.86266	-52076	.85370	-53558	.84448	37
24	-49090	.87121	.50603	.86251	.52101	-85355	.53583 .53607	.84433 .84417	36 35
25	49116	.87107	.50628	.86237	.52126	.85340 .85325	.53632	.84402	34
26	49141	-87093	.50654	.86222 .86207	.52151	.85310	.53656	.84386	33
27 28	·49166	87079	.50679	.86192	.52200	.85294	.53681	.84370	32
20	-49192 -49217	.87064 .87050	.50704	.86178	.52225	.85279	-53705	84355	31
30	49217	.87036	.50754	.86163	+52250	.85264	-53730	.84339	30
31	.40268	.87021	-50779	.86148	.52275	.85249	-53754	.84324	20
32	49203	.87007	.50804	.86133	.52299	85234	-53779	.84308	28
33	.49318	.86993	.50829	.86119	.52324	.85218	.53804	.84292	27
34	49344	.86078	-50854	.86104	-52349	85203	.53828	.84277	26
3.5	49369	.86964	.50879	.86089	-52374	.85188	-53853	.8426I	25
36	•49394	.86949	.50904	.86074	-52399	.85173	-53877	84245	24
37	-49419	86935	.50929	-86059	-52423	.85157	-53902	.84230	23 22
38	-49445	.86921	-50954	.86045	-52448	.85142 .85127	.53926 .53951	.84108	21
30	-49470	.86906	-50979	.86030 .86015	•52473 •52498	.85112	-53975	.84182	20
40	•49495	.86892	.51004		1	.85096	.54000	.84167	10
41	.4952I	.86878	-51029	.86000	-52522	.85090	.54024	.84151	18
42	.49546	.86863	.51054	.85985 .85970	·52547	.85066	.54049	.84135	17
43	49571	.86849 .86834	.51079	.85956	-52597	.85051	-54073	.84120	16
44	.40596 .49622	.86820	.51129	.85941	.52621	.85035	.54007	84104	15
45	49647	.86805	.51154	.85926	.52646	.85020	.54122	84088	14
47	.49672	.86791	.51179	.85011	.52671	.85005	.54146	.84072	13
48	49697	.86777	.51204	.85896	.52696	.84989	-54171	.84057	12
40	.49723	.86762	51229	.85881	.52720	.84974	-54195	.8404T	11
50	.49748	86748	-51254	.85866	-52745	.84959	.54220	.84025	10
5 r	-49773	.86733	.51279	.85851	.52770	.84943	-54244	.84000	9
52	.40798	.86719	.51304	.85836	-52794	.84928	.54269	83994	8
53	.49824	.86704	-51329	.85821	.52819	.84913	-54293	.83978	7 6
54	.49849	.86690	-51354	.85806	-52844	.84897	-54317	83962	
5.5	.40874	.86675	-51379	.85792	.52869	.84882	•54342 •54366	.83946	5 4
50	49899	.86661	.51404	.85777	.52893	84851	-54301	83015	1 2
5.7	.49924	.86646 .86632	.51429 .51454	.85702	-52943	.84836	-54415	83800	3 2
58	.49959	.86617	.51454	.85732	.52967	-84820	.54440	.83883	1
50 60	.49975 .50000	.86603	.51504	.85717	.52992	.84805	-54464	.83867	0
-		-		-	1			Cause	1
•	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	1
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	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSNIE	1
-	.54464	.83867	-55919	.82904	-57358	.81915	.58779	80902	60
r	.54488	.83851	•55943	82887	.57381	.81899 .81882	.58802 .58826	.80885 .80867	59 58
2	•54513 •54537	.83835 .83819	.55968 -55992	.82871 .82855	•57405 •57429	.81865	.58849	.85850	57
3 4	·5456I	.83804	.56016	.82839	•57453	.81848	.58873	-80833	56
5 6	-54586	.83788	.56040	.82822	-57477	81832	.58896	.8081ó	55
6	-54610	.83772 .83756	.56064 .56088	.82806 .82790	-57501 -57524	.81815 .81798	.58920 .58943	.80799 .80782	54 53
7 8	.54635 .54659	.83740	.56112	.82773	.57548	.81782	.58967	-80765	52
9	.54683	.83724	.56136	.82757	-57572	.81765	.58990	-80748	51
10	.54708	83708	•26160	.82741	-57596	.81748	.59014	.80730	50
II	.54732	83692	.56184 .56208	.82724 .82708	.57619 .57643	.81731 .81714	.59037 .59061	.80713 .80696	49 48
12	.54756 .54781	.83676 .83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	-54805	.83645	.56256	82675	-57691	.81681	.59108	-80662	46
15	54829	.83629	.56280	.82659	-57715	.81664 .81647	.59131 .59154	.80644 .80627	45 44
17	.54854 .54878	.83613 .83597	.56305 .56329	.82643 .82626	.57738 .57762	.81631	.59178	.80610	44
18	.54902	.83581	.56353	.82610	.57786	.81614	.5920 I	.80593	42
19	-54927	-83565	-56377	.82593	.57810	.81597 .81580	.59225 .59248	.80576 .80558	41
20	-54951	.83549	.56401	82577	-57833	.81563	.59272	.8054r	
2 I 2 2	•54975 •5499 9	.83533 .83517	.56425 .56449	.82561 .82544	.57857 .57881	.81546	.59272	.80524	39 38
23	-55024	.83501	.56473	.82528	-57994	.81530	.59318	.80507	37
24	.55048	.83485	.56497	82511	.57928	81513	-59342	.80489	36
25 26	.55072 .55097	.83469 .83453	.56521 .56545	.82495 .82478	-57952 -57976	.81496	.59365 .59389	.80472	35 34
27	.55121	.83437	.56569	.82462	.57999	.81462	59412	.80438	33
27 28	-55145	.83421	-56593	.82446	.58023	.81445	.59436	-80420	32
29 30	.55169 .55194	.83405 .83389	.56617 .56641	.82429	.58047	.81428	-59459 -59482	.80403 .80386	31
31	·55218	.83373	.56665	.82396	.58094	.81395	.59506	.80368	20
32	-55242	.83356	.56689	.82340	.58118	.81378	-59529	.80351	28
33	-55266	.83340	-56713	.82363	.58141	.81361	-59552	.80334 .80316	27
34	.55291 .55315	.83324 .83308	.56736	.82347 .82330	.58165	.81344	-59576 -59599	.80299	25
35 36	•55339	.83292	.56784	.82314	.58212	.81310	.59622	.80282	24
37 38	E = 262	.83276	.568o8	.82297	.58236	.81293	.59646	.80264 .80247	23
38 39	.55388 .55412	.83260 .83244	-56832 -56856	.82281 .82264	.58260	.81276	.59669 .59693	.80230	21
40	.55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	-55460	.83212	.56904	.82231	.58330	.81225	-59739	.80195	10
42	-55484	.83195	.56928	.82214	.58354	.81208	-59763	.80178	18
43 44	-55509 -55533	.83179 .83163	.56952 .56976	.82198 .82181	.58378	.81191	.59786 .59809	.80143	17
45	-55557	83147	-57000	.82165	.58425	.81157	.50832	.80125	15
46	-55581	83131	-57024	.82148	-58449	.81140	.59856	80108	14
47 48	.55605 .55630	.83115 .83098	-57047 -57071	.82132	.58472 .58496	.81123	.59879 .59902	.80073	13
49	-55654	.83082	-57095	82098	.58519	.81089	.59926	80056	II
50	.55678	.83066	-57119	-82082	-58543	.81072	-59949	.80038	10
51	-55702	.83050	-57143	.82065	.58567	.81055	-59972	.80021	8
52 53	-55726	.83034 .83017	-57167	.82048 .82032	.58590	.81038 .81021	.59995	.80003	7
54	-55775	.83001	-57191	.82032	.58637	.81004	.60042	.79968	7 6
55 56	-55799	82985	-57238	.81999	.58661	.80987	.60065	·79951	5 4
50 57	.55823 .55847	.82969 .82953	.57262 .57286	.81982 .81965	.58684 .58708	.80970 .80953	.60089	.79934 .79916	3
57 58	.55871	.82936	.57310	.81905	.58731	.80936	.60135	.79899	3 2
59 50	-55895	.82920	-57334	.81932	-58755	.80019	.60158	.79881	1 0
	-55919	-82904	-57358	.81915	-58779	.80902	.00182	.79864	
,	COSINE	SINE	Cosine 5	SINE	Cosine 5	SINE	Cosine 5	SINE	1

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,	SINE	Cosine	SINE	Cosine	Sine	Cosine	SINE	Cosine	
-	.60182	.79864	.61566	.788or	.62932	-77715	.64279	.76604 .76586	60 59
I	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76567	58
2	.60228	-79829	.61612	.78765 .78747	.63000	.77660	.64346	.76548	57
3	.60251	.79811	.61658	.78729	.63022	.7764I	.64368	.76530	56
4	.60274 .60298	.79793	.61681	.78711	.63045	.77623	-64300	.76511	55
5	.60321	.79758	.61704	.78694	.63068	-77605	.64412	.76492	54
7	.60344	-79741	.61726	.78676	.63090	.77586	.64435	.76473	53
7	.60367	-79723	.61749	.78658	63113	.77508	-64457	-76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
ΙÓ	.60414	.79688	.01795	.78622	.63158	·77531	.64501	.76417	50
11	.60437	.79671	81810.	.78604	.63180	·77513	.64524	.76398 .76380	49 48
12	.60460	-79653	.61841 .61804	.78586 .78568	.63203	·77494 ·77476	.64546	.76361	47
13	.60483	.79635 .79618	.61887	.78550	.63248	.77458	.64500	.76342	46
14	.60506 .60529	.79000	.61909	.78532	.63271	.77439	.64612	.76323	45
15 16	.60529	.79583	.61932	.78514	.63293	.77421	.64635	-76304	44
17	.60576	.79505	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	-79547	.61278	.78.178	.63338	-77384	.64679	-76267	42
19	.60622	.79530	.6200I	78460	.63361	.77366	.64701	.76248	4I
20	.60645	.79512	.02024	.78442	.63383	.77347	.64723	.76229	40
21	.60068	-79494	.62046	.78424	.63406	-77329	.64746	.76210	39 38
22	.60691	-79477	.62000	-78495	.63428	.77310 .77292	.64768	.76192 .76173	37
23	.60714	·79459	.62092	.78387 .78369	.63451	.77273	.64812	.76154	36
24	.60738	·79441	.62138	.78351	.63496	.77255	.64834	.76135	35
25	.60761	.79424 .79406	.62160	.78333	.63518	.77236	64856	.76116	34
26	.60807	.79388	.62183	.78315	.63540	.77218	64878	.76007	33
27	.60830	.79371	.62206	.78297	.63563	.77100	.6.1901	.76078	32
20	.60853	-79353	.62220	.78279	.63585	.771St	.64923	-76059	31
30	.00876	-79335	.02251	.78201	.63608	.77162	.64945	.76041	30
31	.60800	.79318	.6227.1	.78243	.63630	-77144	64067	.76022	29 28
32	.60922	.79300	.62297	.78225	.63653	-77125	.64989	-76003 -75984	27
33	.60045	.79282	.62320	.78206 .78188	.63675	.77107 .77088	.65033	•75964 •75965	26
34	.60008	-70264	.02342	.78170	.03720	77070	.05055	-75946	25
35	.61015	.70247	.62388	.78152	.63742	.77051	-65077	-75927	24
36	.61038	.70211	.62411	.78134	.63705	.77033	.05100	-75008	23
37 38	.61061	.70103	.62.433	.78116	.63787	-77014	.05122	-7588o	22
39	.61084	-70176	.62456	.78008	.63810	70096	.65144	-75870	21
40	.61107	.79158	.62479	.78070	.63832	-70977	.65166	-75851	20
41	.61130	.791.10	.62502	.78061	.63854	-76050	.65188	•75832	81
42	.61153	.70122	02524	-780-43	.63877	-70040 -70021	.65210	-75813 -75794	17
43	.61176	-70105	-025-17	.78025	63922	-70003	.65254	•75775	16
44	.61100	-70087	.62570	.78007 .77988	.03044	76884	.65276	75756	15
45 46	.61222	.70060	.02015	.77070	-63000	-76866	.65208	-75738	14
47	.61268	.70033	62638	-77952	.63080	-70847	.65320	-75719	13
48	.61201	70016	.02000	-77934	1101.0.	-76828	.05342	75700	12
49	.61314	.78008	.62683	.77010	.0.1033	.70810	.05304	75080	TI
50	.61337	.78980	.02700	-77897	.64056	-70791	.65386	.75001	10
5 T	.61360	.78062	.62728	-77870	.6.1078	.76772	.65.408	-75642	8
52	61383	-78044	02751	.77801	.0.1100	-70754	.05.130	75023	0
53	.61406	.78020	.02774	-778.13	64123	.76735 .76717	.05.152	-75004 -75585	7 6
54	.61429	.78008	.62706	.77824 .77800	.0.1107	.70008	.05.174	.75506	5
55	.61451	.78891	.02810	-77788	0011.0.	.70070	.05518	-75547	4
56	.61474	.78855	.0280.1	.77700	.0.1212	.70001	.055.10	75528	3 2
57 58	.61520	.78837	.02887	-77751	.6.12.3.4	.700.12	.05502	-75500	
59	.61543	.78810	0.000	-77733	.64250	.76623	.05584	.75.100	1
60	.61566	.78801	.62032	-77715	.0.1270	.7660.1	.05000	-75-171	0
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111111111111111111111111111111111111111	2 .65866 3 .65891 4 .65913 5 .65935 6 .65956 7 .65978 8 .66000	7 .7526 -7524 -7522 -7520 -7518 -7516 -7514 -7512	.6715 1 .6717 2 .6719 3 .6721 4 .6723 5 .6725 6 .6728 6 .6730 7 .67323	2 .7408 4 .7404 7 .7402 8 .7402 0 .7398 1 .7396 3 .7394	30 51 51 52 52 53 4	.6843. .6847. .6847. .68518. .6853. .68563. .68582. .68603.	4 .729 5 .728 6 .723 7 .728 8 .728 9 .728 1 .727 2 .727 3 .727	97 77 57 37 17 97 77	.6969 .6971 .6973 .6977 .6980 .6982 .6984 .6986	06 7 7 8 9 0 1 2 2	.7171 .7160 .7167 .7163 .7161 .7150 .7156 .7154	1 1 0 0 0 0 0 0	40 48 47 46 45 44 43 42 41 40
21 22 23 24 25 26 27 28 29	.66088 .66109 .66131 .66153 .66175	.75060	67366 67387 67499 67498 67452 67473 67495 67516 67538	.7390 .7388 .7386 .7384	5 5 5 7 7 7	.68645 .68666 .68688 .68709 .68730 .68751 .68772 .68793 .68814	7270 -7260 -7265 -7263 -7261 -7259 -7257 -7257	7 7 7 7 7 7 7 7	.6990. .6992 .69940 .69960 .70008 .70020 .70040	4 5555	.71500 -71480 -71460 -7144 -7142 -7140 -71380 -71360 -71345	3 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	39 38 37 36 35 34 33 32 31
31 32 33 34 35 36 37 38 39 40	.66284 .66306 .66327 .66349 .66371 .66393 .66414 .66436 .66458	.74876 .74857 .74838 .74818 .74799 .74780 .74760 .74741 .74722	.67580 .67602 .67623	.73708 .73688 .73669 .73649 .73629 .73570 .73570 .73551		.68857 .68878 .68899 .68920 .68941 .68962 68983 69004 69025	-7253 -7251 -7249 -7247 -7245 -7243 -7241 -7239 -7237 -72357 -72337	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	.70091 .70112 .70132 .70153 .70174 .70195 .70215 .70236 .70257 .70277 .70208		71325 71305 71284 71264 71243 71223 71203 71182 71162 71141		30 20 28 27 20 25 24 23 22 21
41 42 43 44 45 46 47 48 49 50	.66501 .66523 .66545 .66566 .66588 .66610 .66632 .66653 .66675	-74683 -74664 -74644 -74625 -74606 -74586 -74567 -74548 -74528 -74509	.67795 .67816 .67837 .67859 .67880 .67901 .67923 .67944 .67965 .67987	-73511 -73491 -73472 -73452 -73432 -73413 -73393 -73373 -73353 -73353	.0 .0 .0 .0 .0 .0 .0	69067 69088 59109 59130 59151 59172 59193 59214 69235	.7237 .72317 .72297 .72277 .72256 .72236 .72216 .72196 .72176 .72156	1	70319 70339 70360 70381 70401 70422 70443 70463	.7 .7 .7 .7 .7 .7 .7	71121 71100 71080 71039 71039 71019 70998 70978	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
51 52 53 54 55 56 57 58 59	.66718 .66740 .66762 .66783 .66805 .66827 .66848 .66870 .66891	-74489 -74470 -74451 -74431 -74412 -74392 -74373 -74353 -74334 -74314	.68008 .68029 .68051 .68072 .68093 .68115 .68136 .68157 .68179	.73314 .73294 .73274 .73254 .73234 .73215 .73195 .73175 .73155 .73135	.6 .6 .6 .6 .6 .6 .6	9250 9277 9298 9319 9340 9361 9382 9403 9424 9445	.72130 .72116 .72095 .72075 .72055 .72015 .72015 .71995 .71974 .71934	-7 -7 -7 -7 -7 -7 -7 -7	0505 0525 0546 0567 0587 0608 0628 0649 0670 0690	.70 .70 .70 .70 .70 .70	0916 0896 0875 0835 0834 0813 0793 0772 0752 0731		9 8 7 5 4 2 2 1
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1	.00020	3437-750	.01775	55,4500	0,45-21	28 (004	5 6 41,004	\$ 19 79 T 2 2	+ 7
2		1145.020	.01833	54-5013		27 0 174	A16 9 378	39 1459	
3 4	1	859-430	.01802	54.7086	323(6.8)	27.7117	art 157	1 gib 1 d 5 ft gib 1 ft g f	
		687.549	.01801	52.8821	303048	27.4/100	10.00	20 45 41	
5 6	.00175	572.057	.01020	52.0807	11,36,617	27.2715	1115 \$271 1115 \$25	18 1/111	
7 8	.00204	491.100	.01949	50.5485	-03696 -03725	20.5450	115474	24.26. 17	* (
9		381.071	02007	49.8157	413784	261.614617	A155 \$	1911.3	٠,
10	.00201	343.774	.02030	49.1039	0,3775,3	2014,110	417238	\$25 V 1511	,
11	.00320	312.521	.020fiti	48.4121	03812	26.2296	1.16年5年	17 m 1	4
12		280.478	.02005	47-7305		26011117 25 25 (42)	Aintigit	17 (90) 5 17 (90) 6	1
13	.00378	204.441	102124	47.9853	103571	25 2 142	2126121	\$7 7587 6 87 7 11 1	4
14	.00407	245.552	.02153	45.8204	03020	25.4517		37.50	4.5
15 16	.00436	214.858	.02211	45.2201	33,4458	28,2044		37 12 1	4.5
17	.00495	202.210	.02240	44.613861	03057	25 (374)	A12.78		4.3
18	.00524	100.084	.022fkg	44 anteles	opine.	24 76175	217,7562	1 27 1314 1 27 1-19	4 2
19	.00553	180.032	.02208	43.50%2	404-146	34 7185 34-5418	1831 (1885) 1875 (1845)	17 11/0/3	4
20	.00582	171.885	.02,328	45.9044	104044	1	askH+&	10.00	211
21 22	.00611	163,700	02357	42.43.45	334134	24 1075	325,975	Bir grane	5.9
23	.00000	149.405	02415	41-4100	104164		475988	16 9150	9.3
24	.00698	143.237	02444	49.0174		24.44.114	-111948	16.8319	20
25	.00727	137.507	.02473	40.4358	-04770	23 (4)48	113921	\$70 - g. 1870 \$71 - 7111 - 18 2	1 *
26	.00756	1,32,210	.02502	30.00%	374 250	21.5122	arkijega prikorijega	27: 13: 4	2.6
27 28	.00785	127.321	02531	30.5050	234.270 234.49%	24.4727	200 19	27 - 12	2.9
20	.00844	118.540	07,5%0	48.6077	434447	44.0522	18-18-	37 4.89	3.5
30	.00873	114.580	102010	48.1355	it is geies	asapigh.	141391	31 1800	2
31	.00002	110.802	JustiaN.	37.7636	1114 (13%	44.7510	1.14	37 3113	
32	15,000	107.420	.02/177	37-4570	124111	22 la . 3	1918.24	16 19 1	. 9
33	.00000	104.171	012700	36.36.27	1114 15 1	44,4141	1 7 3 B	37 3 1561 37 3 1 4 1 4	
34 35	81010.	08.2170	102764	414.4 7764	114444 E	22.41 74 1 1 22.28/41 1	# 17 #	1000	9.1
36	.010.17	99,4808	102791	45 New Sto		41 11;	18.198	3+ 3-142	9.4
37	.01070	92.0685	makaa	45044444	N1571.		11 5 18	1 - 41 - 1	
38	.01105	88.14 (6	312MK1	the ideas	214 2283	38 7 8 28 6 5 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	13.54.7	X+ 1 1	4.7
39 40	.01135	85.0308	1129111	44-7134	arafradi arafradi		18 9 18 18 18 18 18 18 18 18 18 18 18 18 18	1 1 40	
41	.01103	818449	.02030	14 112 1	1.45.43	22 53514	8.45	3 * * 5 %	
42	.01222	81.8470	i margins !	44 (0) 15 1	314 (27)	28 2 24		4 + 4 3	6
4.3	.01251	70.04 44	(1971)197	\$ 4 4 7 18 1 2	104745	31 11	5 8-01	87	
44	.01280	78.1904	103 (16)	31-1-1	11111	21-13-61-1		x + - 1 - 1 - 8	
45 46	.01300	70.3000	1 314 154 1	高速2.高速度 高速2.高速度	19 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	al Stan alfayra	10 mm g 10 mm g	3	
47	.01307	73.1 10	415214	17.11761	31.3756.3	2 / + fail			
48	.01300	71.611.1	31444	A1 810.	11,276,42	2 - 447-5	1 1		
40	.01425	70.1534		J(\$ 1.3%4)	1. 11.3 2	1 1119		t	
50	.01455	08.7504	133333	41 2411	\$14.214	4		\$ 2.5	
51 52	.01484	67.4010		\$105.00	1.471	1 1 1		1 1 1	
53	.01513	00.1088	angalisi i angalisi i	ξ 3 f.05 ξ 4 ξ1 4 . 4 2 2 f 1	0.7	\$18.54 S	1.44	1.5	
54	01571	Continuen		C + 2 -2 2 fe	1 1 1 1	219 1 2 1		1	
55	.01600	62,4002		744 Linux	******	\$16.5 \$ 75		4	
50	.01620	61.4820	1115 \$ 1961	11/121	17.7.2.5	\$24 + 7 + C		6.41	8
57 58	.01687	(80.40.45	P4415	201 1723	11.31.7	219 2 12	12.	5 4	*
50	.01716	68.264.2	201411 / 201411 / 201411 / 20141 / 201411 / 20141	24 3 22 4	13.13	214 10114	The state of	4 4 9	
60	.01746	\$7.2000	1407	Date Stig	1.242	119 102		1 4	
-	CO-TAN.	17	e					-	
	89 81	T'AN.	COLIAN St	Tan.	tingge. Sylv	1 g 4 c − 2 s 1 c − 2 s	34	. 5	

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1	49	۱) ۲	5°		6	1			,
1	TAN.	CO-TAN.	TAN.	Co-TAN.	TAN.	Co-tan.	TAN.	Co-tan.	
						2 77 426	.12278	8.14435	60
0	.06993	14.3007	.08749	11.4301	.10510	9.51436 9.48781	.12308	8.12481	59
I	.07022	14.2411	.08778	11.3919	.10540	0.46141	.12338	8.10536	58
2	.07051	14.1821	.08807	11.3540	.10509	9.43515	.12367	8.08600	57
3	.07080	14.1235	.08837 .08866	11.3163	.10628	0.40004	.12397	8.06674	56
4	.07110	14.0655	.08805	11.2417	.10657	0.38307	.12426	8.04756	55
5	.07130	14.0079	.08025	11.2048	.10687	9.35724	.12456	8.02848	54
0	.07168	13.9507	.08054	11.1681	.10716	9.33154	.12485	8.00948	53
7 8	.07197	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.00013	11.0954	-10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
		13.6719	.00071	11.0237	.10834	9.23016	.12603	7.93438	49
II I2	.07314	13.6174	.00101	10.9882	.10863	9.20516	.12633	7.91582	48
	.07344	13.5634	.00130	10.9529	.10893	0.18028	.12662	7.89734	47
14	.07373 .07402	13.5098	.09159	10.9178	-10022	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.0746I	13.4039	.09218	10.8483	.10081	9.10646	.12751	7.84242	44
17	07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	07519	13.2996	.09277	10.7797	-11040	9.05789	.12810	7.80622	42
10	.07548	13.2480	.09306	10.7457	-11070	9.03379	.12840	7.78825	41
20	-07578	13.1969	-09335	10.7119	. 11099	9.00983	.12869	7.77035	40
21	-07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39 38
22	.07636	13.0958	-09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.0062	-09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	•07753	12.8081	.09511	10.5136	.11276	8.86862	-13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	·0784I	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	·II394	8.77689	.13165	7-59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	-13195	7.57872	20
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	07958	12.5660	09717	10.2913	.11482	8.70931	13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	-13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	-08046	12.4288	.09805	10.1988	.11570	8.64275	-13343	7.49465	24
37	-08075	12.3838	-09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	-08104	12.3300	.09864	10.1381	.11629	8.57718	.13402	7.44509	21
39	.08134	12.2946	.09893	10.1080	.11688	8.55555	.13432 .13461	7.42871	20
40	.08163	12.2505	.09923		11		11	1	1
41	-08192	12.2067	.09952	10.0483	.11718	8.53402	-13491	7.41240	18
42	-08221	12.1632	.09981	10.0187	.11747	8.51250	.13521	7.39616	17
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	16
44 45	.08300	12.0772	.10040	9.93101	.11836	8.44896	.13500	7.34786	15
45 46	.08339	11.9923	.10009	9.93101	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11805	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11024	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	ır
5Q	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	0
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7 6
54	.08573	11.6645	-10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	-13935	7.17594	4
57 58	.0866r	11.5461	-10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	-10452	9.56791	.12219	8.18370	.13995	7-14553	2
59 60	.08720	11.4685	-10481	9.54106	.12240	8.16398	.14024	7.13042	1
	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	a
7	CO-TAN	TAN.	CO-TAN.	TAN.	Co-tan.	TAN.	CO-TAN.	TAN.	1
	JO TAIN	350 77	JOS-IAN.	84°	CO-IAN.	30 AN.	CU-IAN.	2º TAN.	1
		30	11	O.T.	5	0 1	8	4-	1

	1 8	0	9	0	11 1	.00	11 1	10	1
′	TAN.	CO-TAN.	TAN.	Co-tan.		CO-TAN.		Co-tan.	'
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.T4455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	50
2 3	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19529	5.12069	57 56
5	.14202	7.04105	.15988	6.25486	17783	5.62344	.19589	5.10490	55
	.14232	7.02637	.16017	6.24321	.17813	5.61397	19619	5.00704	54
7 8	.14262	7.01174	.16047	6.23160	.17843	5.60452	19649	5.08921	53
9	.14291	6.98268	.16107	6.22003	.17873	5.59511	.19680	5.08139	52
10	.14351	6.96823	.16137	6.19703	.17903	5.58573	.19710	5.07360	51 50
11	.14381	6.95385	.16167	6.18550	.17963	5.56706	.10770	5.05800	1 -
12	.14410	6.93952	.16106	6.17419	17903		.19801	5.05037	49 48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	40
15	.14499	6.89688	.16286	6.14023	18083	5.53007	.19891	5.02734	4.5
16	.14529	6.88278	.16316	6.12899	.18113	5.52000	-19921	5.01971	44
18	.14588	6.85475	.16376	6.10664	18173	5.51176	.19952	5.01210	43
19	.14018	6.84082	.16405	6.09552	.18203	5.49356	.20012	5.00451 4.99095	42 41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5-47548	.20073	4.08188	39
22	.14707	6.79936	.16495	0.00240	.18293	5.46648	-20103	4.97438	38
23	.14737	6.78504	.16525	6.05143	.18323	5.45751 5.44857	.20133	4.96690	37
24	.14767	6.75838	.16555	6.04051	-18353	5.44857	.20104	4.95945	36
25 26	.14826	6.74483	.16615	6.01878	.18383	5.43966	.20194	4.95201	35
27	.14856	6.73133	.16645	6.00797	.18444	5.43077 5.42192	.20224	4.94460	34
28	.14886	6.71789	.16674	5.99720	.18474	5.41300	.20285	4.02084	33
29	-14915	6.70450	.10704	5.98046	18504	5.40429	.20315	4.92249	31
30	-14945	6.69116	.16734	5.97576	.18534	5.39552	-20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	18564	5.38677	-20376	4.90785	20
32	.15005	6.00403	.16794	5.95448	18594	5.37805	-20406	4.00056	28
33 34	.15034	6.63831	.16824	5.94390	.18024	5.36936	-20436 -20466	4.89330	27
35	.15004	6.62523	.16884	5.02283	.18684	5.35206	-20400	4.83005	26
36	15124	6.61219	.16914	5.91235	.18714	5-34345	-20527	4.87162	24
37	.15153	0.50021	.16944	5.00191	18745	5.33487	-20557	4.86444	23
38	.15183	6.58627	.16074	5.80151	.18775	5.32631	-20588	4.85727	22
39 40	.15213	6.50055	.17004	5.88114	.18835	5-31778	.20618 .20648	4.85013	21
41	.15272			5.86051	.18865	5.30928		4.84300	20
42	.15302	6.54777	.17003	5.85024	.18895	5.30080	-20079	4.83500	18
43	.15332	6.52234	.17123	5.84001	.18025	5.28393	-20730	4.82175	17
44	.15302	6.50070	.17153	5.82982	.18055	5-27553	-20770	4.81471	īά
45	.15391	6.40710	.17183	5.81966	.18080	5.26715	-20800	4.80760	15
40	.15421	0.48456	.17213	5.80053	.19016	5.25880	-20830	4.80068	14
47 48	.15451	6.47206	.17243	5.79944 5.78038	19046	5.25048 5.24218	-20801	4.79370	13
49	.15511	6.44720	.17303	5.77036	00101.	5.23391	.20031	4.77978	11
50	.15540	6.43484	-17333	5.70037	.19136	5.22566	.20052	4.77286	10
51	.15570	6.42253	.17363	5-75941	.10166	5-21744	.20082	4.76595	
52	.15000	6.41036	.17393	5-74949	.19197	5.20025	-21013	4.75906	8
5.3	-15030	0.30804	.17423	5.73000	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	-17453	5.72974	-19257	5-10203	.21073	4.74534	6
55 56	.15719	6.36165	.17483	5.71992 5.71013	.19287	5.18480 5.17071	-21104	4.73851	5
57	15749	0.34001	17543	5.70037	.19347	5.16863	.21134	4.73170	4
58	-15779	6.33761	17573	5.00004	.19378	5.10058	-21105	4.71813	2
50	.15800	6.32566	.17003	5.68004	.10408	5.15256	.21225	4.71137	r
60	.15838	0.31375	.17033	5.67128	-19438	5-14455	.21256	4.70403	0
1	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN,	,
	8	lo	80)0 -	7	0°	7	80	

	1 1	.2°	11 1	30		14°	11	15°	1
,	TAN.	CO-TAN.	TAN.	Co-tan.	TAN.	CO-TAN	. TAN.	CO-TAN.	1
-	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
Ī	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	-21316	4.69121	.23148	4.32001	-24995	4.00086		3.72338	52
3	-21347	4.68452	.23179	4.31430	.25026	3.99592		3.71907	5 2
4	-21377	4.67786	.23200	4.30860	.25056	3.99099	.26920	3.71470	50
5 6	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
	.21438	4.66458	.23271	4.29724	-25118	3.98117	.26982	3.70616	54
7 8	.21469	4.65797	.23301	4-29159	-25149	3.97627		3.70100	53
	.21499	4.65138	-23332	4.28595	.25180	3.96651		3.69335	51
9 10	.21529	4.64480	23363	4.27471	.25242	3.96165		3.68909	50
	1	4.63825	11		11	3.95680		3.68485	49
11	.21590	4.63171	-23424	4.26911	.25273	3.95196		3.68061	48
12	.21621	4.62518	-23455 -23485	4.26352	25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23405	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	-23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	-21743	4.59927	.23578	4.24132	.25428	3.93271	-27294	3.66376	44
17	-21773	4.59283	.23608	4.23580	-25459	3.92793	.27326	3.65957	43
18	.21773 .21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	-25552	3.91364	-27419	3.64705	40
31	-21895	4.56726	.23731	4.21387	.25583	3.90890	-27451	3.64289	39
22	-21925	4.56001	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	-23793	4.20298	-25645	3.89945	.27513	3.63461	37
24	-21986	4.54826	.23823	4.19756	.25676	3.89474	-27545	3.63048	36
25	-22017	4.54196	-23854	4.10215	-25707	3.89004	.27576	3.62224	35
26	-22047	4.53568	.23885	4.18675	-25738	3.88536	.27638	3.61814	34
27 28	.22078	4.52941	.23916	4.18137	.25769	3.87601	.27670	3.61405	32
29	.22130	4.52316	.23946	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22160	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745		3.59775	28
33	.22261	4.49215	.24100	4.14934	-25955	3.85284	.27795	3.59370	27 26
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	
3.5	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	-22353	4-47374	-24193	4.13350	.26048	3.83906	.27920	3.58160	24
37	-22383	4:46764	-24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4-46155	-24254	4.12301	.26110	3.82992	.27983	3.57357	22 21
39 40	-22444	4.45548	.24285	4.11778	.26141	3.82537	.28046	3.56957 3.56557	20
	-22475	4-44942	-24316	4.11256	1	1	1)	1	1
41 42	-22505	4.44338	-24347	4.10736	.26203	3.81630	.28077	3.56159 3.55761	18
43	.22536	4-43735 4-43134	·24377 ·24408	4.10216	.26235	3.81177	.28140	3.55364	17
44	.22597	4-43134	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47 48	.22689	4-40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
	.22719	4.40152	-24562	4.07127	.26421	3.78485	-28297	3.53393	12
49	.22750	4.30560	•24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	+24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	8
52	.22842	4-37793	.24686	4.05002	.26546	3.76700	.28423	3.51829	8
53 54	.22072	4.37207	-24717	4.04586	.26577	3.76268	.28454	3.51441	7 6
	.22903	4.36623 4.36040	-24747	4.04081	.26608	3.75828	.28486	3.51053	0
55 56	-22064	4.35459	-24778 -24800	4.03578	.26670	3.75388	.28517	3.50666	5 4
57	-22995	4.34879	.24840	4.03075	.26701	3.74950 3.74512	.28580	3.50279	4 2
57 58	-23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	3
59	-23056	4-33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	ī
бо	-23087	4-33148	-24933	4.01078	.26795	3.73205	.28675	3.48741	o
7	CO-TAN.	Tax	Comus	There			<u> </u>		,
	7	TAN.	CO-TAN. 70	TAN.	CO-TAN.	TAN.	Co-TAN.	1° TAN.	•
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	-06						·		CO-TA.			1	
	0 .286			.3057			.324		3.0770			2.004	
	2 .287			.3000; .3063;			.325		3.0740	1 -34		2.001.	
	3 .287			3000			-325 -325	50	3.0710			2.898;	
	4 .288		216 .	30700			.326.	21	3.0055			2.8032	
	5 .288			30732			-3269	5.3	3.0025.		oti	2.8000	9 99
	7 .288			30764			.3208		3.05050			2.5878	
	7 .288			30796 30828			-3271		3.05040			2.884.1	1 53
	9 .289	58 3.45		30800			.3278		3.05340		.6	2.8824 2.8707	
	10 .2899	3.44	251 .	30801			3.81	4	3.047.40			2.877	
	11 .200:	1 0 - 1-7.		30923			.328.1		3.04450	11		2.874.9	1
	2 .200			30055	3.230.	48	3287		3.0.1152	1148	4.1	2.8716.	1 48
	4 .2011			30987	3.227	5	-3201		3.03854			2,8680.	47
	5 .2914			31019	3.223		3294		3.03556			23600	
	6 .2917			1083	3.2172		•3297; •3300		3.03260 3.02963	+340 -340		ునర్యుగ్ర జనీరియేం	4 .
	7 .2921			1115	3.2130		.3.30.10		3.02667	-3408		2.84822	
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20				1178	3.2073		.3310.		3.02077	-3505		2.85280	41
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2:			, II -2	1274	3.2007		-3316c		3.01480	.3511		2.83748	
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24			71 3	1338	3.1010	0	-33200		110001	-35.21		2.5 toris	17
25	-2940			1370	3.1877	5	-33208	3	01500	3524		2.8 47.12	35
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28	29558			1400	3.1780		•33303 •33395		.007.38	+35.31.		284176	.4.4
29			5 .31	1498	3.17481	. (1	33427		.00447 .00158	+35346		а Кирга. 2 Кирга.	3.7
30	1 -	1 0.0.	. 11	530	3.17150		33400		98868	41412		2.82301	30
31 32				562	3.16838		33402	2.	08;80	35435	í	82111	20
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3.1		3.3015		626 658	3.10197		33557		OSHA	45520		Asterio	27
3.5	-29780			600	3-15558		3358o 33021		07717	-35543 -35556		Margar	211
30 37	.20811	3.3544	3 -31	722	3-15240	11.	33054		071.14		13	Maring.	21
38	29875	3-3508		754	3-14022	11 .	33080	2.	06848	45641		Section 1	51
39	.20000	3-3437	7 31	786	3.14605		33718		0087.4	47074		Sougele	21
40	-29938	3.3.102			3.13072		33751 33783		00255	45797		A	21
41	.20070	3.33070]	3.13656		3816	l		455749	1	34325 1.2	2.1
42	.30001	3.3331	7 -310	11.4	3-13331		(8.48			- 4500 A		705 (S.) 79 250 (11)
43	-30033 -30005	3-32009			3-13027		3881			ALK KAL		19-11	175
4.5	.30007	3.3201.			3.12713		3013			5.0.1		A	ili
40	-30128	3.3191.			3.12400 3.12087		3045			0.00	1 2	N. 24 1	4%
47 48	.30160	3.31509	3.20	74	3-11775		4010			States	1	, Alter	3 \$
40	30102	3.31216			3.11.16.4		4043			ودههای دور داوی		, 3 14 5 , , , (6.1	11
50	.30255	3.30868			3.11153		4973	2.0	uni .	10 (5			11
5 r	30287	3.30174	11		3.108.12	"	4208			to deli	2	(25)	11
52	30310	3.20820	322		3.10532		41.40			(Cit- 1	: د		()
53	-30351	3-20483	.322		5.0001.1		1201		26 (2)	201.14		fri i i	Н
54 55	.30382	3-20130	.3220	00	3.00000		0.8	20	ARA () . Augle () .	ditter.		6403	7
50	-30414 -30440	3.28705	•3•3.	31 3	3.00208		1270	20	1700	30.23		Grago. Tapqo	6
57	30478	3.28442	-3230 -3230		SoSoot	+.3	1993	2.0	15/3/4/1	dere.		152.49	4
58	-30500	3-27767	324		5.08683 5.08370		13.15			Strange	2 /	5 \$195 1	1
50	30541	3.27420	-3246		08073		1400			360 11 1	<i>-</i> 2 .	1711	3
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1	CO-TAN.	TAN.	Co-TA		Tan	- ,	· · ·					1.15.	13
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'	TAN.	Co-tan.	TAN.	CO-TAN.	I AN.	CO-TAN.	TAN.		
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
I	.36430	2.74499	.38420	2.60283	-40436	2.47302	.42482	2-35395	59 58
2	.36463	2.74251	-38453	2.60057	.40470	2.47005	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	-40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572 .40606	2.46476 2.46270	.42619	2.34447	55 54
6	.36595	2.73263	.38587	2.59156 2.58932	.40640	2.46065	.42654	2.34258	53
7 8	.36628	2.73017	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
	.36661	2.72771	.38687	2.58484	.40707	2.45655	.42757	2.3388I	51
9	.36694	2.72281	.38721	2.58261	.40741	2.45451	.4279I	2.33693	50
10			.38754	2.58038	.40775	2.45246	.42826	2.33505	49
II	.36760	2.72036	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
12	.36793 .36826	2.71792	.38821	2.57593	.40843	2.44839	.42804	2.33130	47
13	.36859	2.71305	.38854	2.57371	.40877	2.44636	-42020	2.32943	46
14	.36892	2.71062	.38888	2.57150	.40011	2.44433	.42963	2.32756	45
15	.36925	2.70819	.38921	2.56928	-40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	•38955 •38988	2.56707	-40979	2.44027	.43032	2.32383	43
18	.36991	2.70335		2.56487	.41013	2.43825	.43067	2.32197	42
IQ	.37024	2.70094	.39022	2.56266	-41047	2.43623	.43101	2.32012	41
20	-37057	2.69853	-39055	2.56046	.41081	2.43422	.43136		40
21	.37090	2.69612	.39089	2.55827	-41115	2.43220	-43170	2.31641	39
22	.37124	2.69371	-39122	2.55608	-41149	2.43019	.43205	2.31456	38
23	-37157	2.69131	.39156	2.55389	.41183	2.42810	-43239	2.31271	37 36
24	.37190	2.68892	-39190	2.55170	-41217	2.42618	.43274	2.30902	35
25 26	.37223	2.68653	-39223	2.54952	.41251	2.42418	.43308 .43343	2.30718	34
	.37256	2.68414	•39257 •39290	2.54734	.41319	2.42019	.43378	2.30534	33
27 28	.37289	2.67937	-39324	2.54299	.41353	2.41819	.43412	2.30351	32
	.37322	2.67700	-39357	2.54082	.41387	2.41620	-43447	2.30167	31
29 30	·37355 ·37388	2.67462	-39391	2.53865	.41421	2.41421	-43481	2.29984	30
-		2.67225	-39425	2.53648	-41455	2.41223	.43516	2.29801	29
31	.37422	2.66989	-39458	2.53432	-41490	2.41025	-43550	2.20619	28
32 33	·37455 ·37488	2.66752	-39492	2.53217	-41524	2.40827	-43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	-43620	2.29254	26
35	-37554	2.66281	-39559	2.52786	-41592	2.40432	-43654	2.20073	25
36	·37554 ·37588	2.66046	-39593	2.52571	-41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	-39626	2.52357	-41660	2.40038	·43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	-43758	2.28348	21
39	.37687	2.65342	.39694 .39727	2.51929	.41728 .41763	2.39449	-43793 -43828	2.28167	20
40	.37720							2.27087	
41	-37754	2.64875	-39761	2.51502	.41797	2.39253	.43862 .43897	2.27806	18
42	-37787	2.64642	-39795 -39829	2.51289 2.51076	.41865	2.38862	·43097 ·43932	2.27626	17
43	-37820 -37853	2.64410	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
44	-37387	2.63945	.39896	2.50652	·41933	2.38473	.44001	2.27267	15
45 46	.37920	2.63714	-39930	2.50440	.41068	2.38279	.44036	2.27088	14
47	.27052	2.63483	-39963	2.50229	-42002	2.38084	.44071	2.26909	13
47 48	.37986	2.63252	-39997	2.50018	-42036	2.37891	.44105	2.26730	12
49	.35020	2.63021	-40031	2.49807	-42070	2.37697	-44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	-44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	8
52	.38120	2.62332	-40132	2.49177	.42173	2.37118	-44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	-42207	2.36925	-44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242 .42276	2.36733	-44314	2.25486	
55 56	.38220	2.61418	.40234 .40267	2.48340	.42310	2.36349	·44349 ·44384	2.25300	5 4
57	.38286	2.61100	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
57 58	.38320	2.60963	-40335	2.47924	.42379	2.35967		2.24956	2
50	.38353	2.60736	.40369	2.47716	.42413	2.35776	-44453 -44488	2.24780	1
60	.38353 .38386	2.60509	.40403	2.47509	-42447	2.35585	-44523	2.24604	0
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2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
2.2.4.428	*10000	2.14288	.48809	2.04879	.50989	1.96120	59
2.2.4252	-40702 -40737	2.14125	.48845 .48881	2.04728	.51026 .51063	1.95979	58
2.23002	-40772	2.13801	.48017	2.045//	.51003	1.95838	57 56
2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
2.23553	.408.13	2.13477	.48989	2.04125	.51173	1.95417	54
2.23378	40879	2.13316	-49026	2.03975	.51200	1.95277	53
2.23204	40014	2.13154	49062	2.03825	-51246	1.95137	52
2.23030	-40050 -40985	2.12003	.49134	2.03675	.51283	1.94097 1.94858	20 21
2.22083	.47021	2.12671	49170	2.03376	-51356	1.94718	49
2.22510	-47056	2.12511	(1)20()	2.03227	·51393	1.94579	48
2.22337	.47002	2.12350	.40242 .40278	2.03078	.51430 .51407	1.94440	47 46
2.21002	.47163	2.12030	.49315	2.02780	.51503	1.94301	45
2.21810	-47199	2.11871	-4035I	2.02031	.51540	1.94023	44
2.21047	-17234	2.11711	-49387	2.02483	-51577	1.93885	43
2.21475	17270	2.11552	-40423	2.02335	.51614	1.03746	42
2.2130.4	-47305	2.11302	-49459	2.02187	·51651	1.93608	4 I
2,21132	-47341 -47377	2.11233	-49495 -49532	2.02039	.51688	1.93470	40
2.20700	47412	2.10010	.40568	2.017.13	.51761	1.93332	39 38
2.20010	-47448	2.10758	-40004	2.01500	.51798	1.93057	37
2.20440	-47483	2.10000	.40040	2.01449	-51835	1.92920	36
2.20278	-47519	2.10.1.12	-19077	2.01302	.51872	1.92782	35
2.20108	-17555	2.10284	-19713	2.01155	.51900	1.02645	34
2.10038	-47500	2.10120	-49749	2.01008	.51046	1.92508	3.3
2.10700	-47626 -47662	2.00000	.40786	2.00862	.51983 .52020	1.92371	32
2.19430	.47098	2.09054	.49858	2.00569	-52057	1.92235	31 30
2.10261	-47733	2.00408	-49894	2.00423	-52004	1.01062	20
2.18023	-47700 -47805	2.00341	(0031	2.00277	-52131	1.91826	28
2.18755	.478.40	2.00028	.40007	1.00086	.52108	1.91690	27
2.18587	.47876	2.08872	.500.10	1.00841	-52242	1.01418	25
2.18410	.47012	2.08716	50076	1.00005	-52270	1.01282	24
2.18251	47048	2.08500	.50113	1.00550	-52316	1.011.17	2,3
2.18084	.47084	2.08.105	.50140	1.00400	-52353	1.01012	22
2.17016	.48055	2.08250	.50185	1.00201	-52,300	1.00876	21
2.17740	.48001	2.07030	.50258	1.08072	-52427 -52464	1.00007	10
2.17410	.48127	2.07785	.50205	1.08828	52501	1.00472	18
2.17240	.48103	2.07030	-50331	1.08084	52538	1.00337	17
2.17083	.48198	2.07.170	-50368	1.08540	-52575	1.00203	16
2.10017	-48234	2.07321	-50404	1.08306	52013	1.00060	15
2.16589	.48270	2.07167	-50441	1.08253	.52050	1.80935	1.1
2.10585	-48306 -48342	2.07014	-50477	1.07000	.52687 .52724	1.80667	13
2.16755	18378	2.00700	.50514	1.07823	.52761	1.80533	11
2.16000	.48414	2.00553	.50587	1.97080	.52708	1.80400	10
2.15025	.48.150	2.00,100	-50623	1.07538	.52836	1.80266	8
2.15700	.48480	2.00247	-50000	1.07305	52873	1.80133	
2.15500	.48521	2.00004	.50000	1.07253	.52010	1.88867	7
2.15432	-48557 -48503	2.05042	-50733 -50700	1.07111	.52047 .52084	1.88734	5
2.15104	48020	2.05037	50800	1.008.27	.53022	1.88002	4
2.14040	.48665	2.05.185	508.13	1.00685	.53050	1.88460	3
2.14777	18701	2.05333	.50870	1.00544	-53006	1.88337	2
2.1401.4	-48737	2.05182	50010	1.00.102	-53134	1.88205	1
2.14451	18773	2.05030	.50053	1.06261	-53171	1.88073	0
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50 1 1.3.	11 6	.[0	li 6	30	11 6	20	1

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	TAN.	Co-TAN.	TAN.	Co-tan	. TAN.	CO-TAN.	TAN.	Co-TAN.	
0	.53171	1.88073 1.87941 1.87809	.55431 .55469 .55507	1.80405 1.80281 1.80158	-57774	1.73205 1.73089 1.72973	.60086 .60126 .60165	1.66428 1.66318 1. 6209	50 58
2 3 4	.53246 .53283 .53320	1.87677	·55545 ·55583	1.80034	.57851	1.72857	.60205 .60245	1.66099	57
5 6	.53358 .53395	1.87415	.55621	1.79788	-57929 -57968	1.72625	.60284 .60324	1.65881	55 54
7	-53432 -53470	1.87152	.55697 .55736	1.79542	-58007 -58046	1.72393	.60364	1.65663	53 52
9 10	.53507 .53545	1.86891	·55774 ·55812	1.79296 1.79174	.58085	1.72163	.60443	1.65445	51
11 12	.53582 .53620	1.86630	.55850	1.78029	.58162	1.71932	.60522	1.65228	49 48
13	.53657 .53694	1.86369	.55926 .55964	1.78807	.58240 .58270	1.71702	.60602	1.65011	47 46
16	·53732 ·53769	1.85109	.56003 .56041 .56079	1.78563 1.78441 1.78319	.58318 .58357 .58396	1.71473 1.71358 1.71244	.60681 .60721 .60761	1.64795 1.64687 1.64579	45 44 43
17 18 19	.53807 .53844 .53882	1.85720	.56117	1.78198	.58435 .58474	1.71129	.60801	1.64471	42 41
20	.53920	1.85462	-56194	1.77955	-58513	1.70901	.60881	1.64256	40 39
2I 22 23	•53957 •53995 •54032	1.85333 1.85204 1.85075	.56232 .56270 .56309	1.77713	.58552 .58591 .58631	1.70673	.60960	1.64041	38 37
24 25	-54070 -54107	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36 35
26 27	.54145 .54183	1.84689	.56424 .56462	1.77230	.58748 .58787	1.70210 1.70106	.61120	1.63612 1.63505	34 33
28 29	-54220 -54258	1.84433	.56500 .56539	1.76090	.58826 .58865	1.69992	.61200	1.63398 1.63292	32 31
30	.54296 .54333	1.84177	.56577 :56616	1.76749	.58904 .58944	1.69766	.61280	1.63185	30 29
32 33	•54371 •54409	1.83922 1.83794	•56654 •56693	1.76510	.58983 .59022	1.69541	.61360	1.62072	28 27
34 35 36	.54446 .54484	1.83667	.56731 .56769	1.76271	.59061 .59101	1.69316	.61440	1.62760 1.62654	26 25
37	•54522 •54560	1.83413	-56808 -56846	1.75032	.59140 .59179	1.69091	.61520	1.62548	24 23
38 39	•54597 •54635 •54673	1.83159 1.83033 1.82906	.56885 .56923 .56962	1.75794	-59218 -59258	1.68866	.61641	1.62336	22 21
40	-54711	1.82780	.57000	1.75556	-59297 -59336	1.68643	.61681	1.62125	10
42 43 44	-54748 -54786 -54824	1.82654 1.82528 1.82402	.57030 .57078	1.75319	-59376 -59415	1.68419	.61761	1.61914	18
45	-54862 -54900	1.82276	.57116 .57155 .57193	1.75082 1.74964 1.74846	•59454 •59494	1.68196 1.68085 1.67074	.61842 .61882 .61922	1.61703	16 15 14
47 48	·54938 ·54975	1.82025	.57232 .57271	1.74728	.59533 .59573 .59612	1.67863	.61962	1.61403 1.61388 1.61283	13
49 50	.55013 .55051	1.81774	·57300 ·57348	I.74492 I.74375	.59651 .59691	1.67641	.62043	1.61179	11
51 52	.55089 .55127	1.81524	.57386 .57425	1.74257 1.74140	•59730 •59770	1.67419	.62124 .62164	r.60070 r.60865	9
53 54	-55165 -55203	1.81274	·57464 ·57503	1.74022	.59809 .59849	1.67108	.62204 .62245	1.60761	7
55 56	-55241 -55279	1.81025	·57541 ·57580	1.73788	.59888 .59928	1.66978	.62285	1.60553	5 4
57 58	-55317 -55355	1.80777	.57619 .57657	1.73555	.59967 .600.57	1.66647	.62366	1.60345	3
59	·55393 ·55431	1.80529	.57696 .57735	1.73321 1.73205	.60046 .60086	1.66538	.62446 .62487	1.60033	a a
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,	TAN.	CO-TAN.	TAN.	Co-tan.	TAN.	Co-tan.	TAN.	Co-tan.	,
-	.62487	1.60033	.64941	1.53986	.67451	1.48256	-7002I	1.42815	60
ī	.62527	1.59930	.64982	1.53888	.67493	1.48163	-70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	-70107	1.42638	58
	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	
3	.62649		.65106		.67620	1.47885	-70194	1.42462	57 56
4		1.59620	677.48	1.53595					
5	.62689	1.59517	.65148	I.53497	.67663	1.47792	-70238	1.42374	55
0	.62730	1.59414	.65189	1.53400	.67705	1.47699	-70281	1.42286	54
7 8	62770	1.59311	.65231	1.53302	.67748	1.47607	-70325	1.42198	53
	62811	1.59208	65272	1.53205	.67790	1.47514	-70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	62892	1.59002	.65355	1.53010	.67875	1.47330	-70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	43
13	63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045 .68088	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	. 63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
17	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51046	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70079	1.40887	33
23	.63421	1.57676	.65806	1.51754	-68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	30
25	.63503	1.57474	.65980	1.51562	.68514		.71110	1.40627	35
26	62544	1.57372	.66021	1.51466	.68557	1.45955	.71154	1.40540	34
	.63544 .63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
27 28	63625	1.57170	.66105	1.51275	68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66180	1.51084	.68728	1.45501	.71329	1.40195	30
	1								1
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	·71373	1.40109	29 28
32	63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27 20
34	.6337I	1.56566	.66356	1.50702	.68900	1.45139	-71505	1.39850	
35	.63012	1.56466	.66398	1.50607	.68942	1.45049	-71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37 38	63094	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.04035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.30421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71760	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	10
42	64199	1.55766	*66692	1.49944	.69243	1.44418	.71857	1.39165	13
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	-71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	-69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	-72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	II
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	-72255	1.38399	98
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	
53	.64652	1.54675	.67155	1.48900	.69718	1.43436	-72344	1.38229	7 6
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	-72477	1.37976	4
	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
57 58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
59 60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72054	1.37638	0
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•	TAN.	CO-TAN.		Co-tan.	TAN.	Co-tan.	TAN.	Co-TAN.	_
0	.72654	1.37638	·75355	1.32704	.78129	1.27994	.80978	1.23490	бo
1	.72699	1.37554	.75401	1.32624	-78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	•75447	1.32544	.78222	1.27841	.81075	1.23343	58
	.72788	1.37386	-75492	1.32464	.78269	1.27764	.81123	1.23270	57
3	.72832	1.37302	.75538	1.32384	.78316	1.27688	81171	1.23196	50
4	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
5	·72021	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
~	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	5.3
7 8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22004	52
		1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
9	-73055 -73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
13				1.31825	.78645	1.27153	.81510	1.22685	40
11	.73144	1.36716	.75858		.78692	1.27077	.81558	1.22012	48
12	.73189	1.36633	.75904	1.31745	.78739	1.27001	.81606	1.22530	47
13	-73234	1.36549	·75950	1.31666	.78786	1.20025	.81655	1.22407	40
14	.73278	1.36466	.75996	1.31586	.78834	1.26840	.81703	1.22394	45
15	·73323	1.36383	.76042	1.31507	.78881	1.20774	.81752	1.22321	44
16	.73368	1.36300	.76088	1.31427	.78928	1.26698	.81800	1.222.19	43
17	·734I3	1.36217	.76134	1.31348	-78975	1.26622	.81849	1.22176	42
18	·73457	1.36133	.76180	1.31269		1.26546	.81898	1.22104	41
19	-73502	1.36051	.76226	1.31190	.79022	1.26471	.81946	1.22031	40
20	•73547	1.35968	.76272	1.31110	.79070		1	-	
21	-73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	30
22	-73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	-73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	-79259	1.26169	.82141	1.21742	30
	-73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
25 26	-73816	1.35472	.76548	1.30637	-79354	1.26018	.82238	1.21598	34
27	-7386I	1.35389	.76594	1.30558	.79401	1.25043	.82287	1.21526	3.3
28	-73906	1.35307	.76640	1.30480	-79449	1.25067	.82336	1.21454	32
20	•7395I	1.35224	.76686	1.30401	-79496	1. 25702	.82385	1.21382	31
30	•73996	1.35142	.76733	1.30323	-79544	1.25717	.82434	1.21310	30
		1.35060	.76779	1.30244	-7959I	1.25642	82483	1.21238	20
31	.74041 .74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21100	28
32	.7413I	1.34896	.76871	1.30087	.79686	1.25402	.82580	1.21004	27
33	.74176	1.34814	.76018	1.30000	.79734	1.25417	.82020	1.21023	20
34	.74221	1.34732	.76964	1.20031	.79781	1.25343	.82678	1.20051	25
35 36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	2.1
	.74312	1.34568	77057	1.29775	.79877	1.25193	.82776	1.20808	2.3
37 38	•74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20005	21
40	-74447	1.34323	.77196	1.20541	.80020	1.24900	.82923	1.20593	20
		1			.80067	1.24805	.82072	1.20522	10
41	•74492	1.34242	.77242	1.29463	.80115	1.24820	.83022	1.20451	18
42	-74538	1.34160	.77289		.80163	1.24746	.83071	1.20370	17
43	-74583	1.34070	•77335	1.20220	.80211	1.24672	.83120	1.20308	10
44	-74628	1.33998	.77382	1.20152	.80258	1.24597	.83160	1.20237	15
45	-74674	1.33916	.77428	1.29074	.80306	1.24523	.83218	1.20100	1.1
46	.74719	1.33835	-77475	1.280074	.80354	1.24449	.83268	1.20005	1,3
47 48	•74764	1.33754	.77521	1.28010	.80402	1.24375	.83317	1.20024	1.2
	.74810 .74855	1.33673	.77568 .77615	1.28842	.80450	1.24301	.83366	1.10053	11
49 50	•74900	1.33592	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
						- 1			
51	-74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.1801.1	8
52	•74991	1.33349	-77754	1.28610	.80594	1.24079	.83514	1.10740	
53	-75037	1.33268	-77801	1.28533	.80642	1.24005	.83564	1.10000	7
54	.75082	1.33187	.77848	1.28456	.80000	1.23031	.83613	1.10500	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.10528	- 5
56	·75173	1.33026	.7794t	1.28302	.80786	1.23784	.83712	1.10457	-4
57 58	-75219	1.32046	.77988	1.28225	.80834	1.23710	.83761	1.10387	- 3
58	-75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.10316	2
59 60	-75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.10240	1
00	·75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	Q
7	CO-TAN.	TAN.	Co-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN	,
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-	TAN.	CO-TAN.	TAN.	Co-tan.	TAN.	Co-tan.	TAN.	Co-tan.	_
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	бо
I	.83960	1.19105	.86980	1.14969	.90093	1.10996	-93300	1.07174	59
2	.84000	1.19035	.87031	1.14902	.90146	1.10031	.93360	1.07112	58
3	.84050	1.18064	.87082	1.14834	.90199	1.10867	.93415 .93469	1.05049	57 50
4	.84108	1.18894	.87133	1.14767	.90251	1.10002	-93524	1.00925	55
5	-84158	1.18824	.87184 .87236	1.14632	.90357	1.10072	93578	1.06862	54
	.84208 .84258	1.18684	.87287	1.14565	.90410	1.10007	-93633	1.06800	53
7 8	.84307	1.18614	.87338	1.14408	.90.163	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87380	1.14430	.90516	1.10478	-93742	1.06676	51
10	.84407	1.18474	.87441	1.14303	.90569	1.10414	-93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	40
12	.84507	1.18334	.87543	1.14220	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14005	.90781	1.10156	.04016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94125	1.06241	45 44
16	.84706	1.18055	.87749 .87801	1.13961	.90940	1.000063	.94180	1.06170	43
17	.84756 .84806	1.17986	.87852	1.13828	.00003	1.00800	.94235	1.06117	42
18	.84856	1.17846	87004	1.13761	.01046	1.00834	.94290	1.00056	41
20	.84906	1.17777	87055	1.13094	.01000	1.09770	-94345	1.05994	40
- 1	*84956	1.17708	.88007	1.13627	.01153	1.00706	.94400	1.05032	39
21	.85006	1.17038	.88059	1.13561	.01200	1.00042	-94455	1.05870	38
23	.85057	1.17500	.88110	1.13494	.91259	1.00578	.94510	1.05800	37
24	.85107	1.17500	.88162	1.13428	.01313	1.00514	-94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.01300	1.00450	.04020	1.05685	35
26	.85207	1.17301	.88265	1.13205	.91419	1.00386	-94676	1.05624	34
27	85257	1.17202	.88317	1.13228	.91473	1.00322	-94731 -94786	1.05501	33
28	.85307	1.17223	.88369 .88421	1.13162	.01520	1.00250	.0484 r	1.05430	31
20	85358	1.17154	.88473	1.13020	.91633	1.09131	-94896	1.05378	30
30	.85.108	1	.88524	1.12063	.01687	1.00067	04952	1.05317	20
3 I	.85458	1.17016	.88576	1.12003	.01740	1.00003	.05007	1.05255	28
32	.85509 .85559	1.16947	.88628	1.12831	-01704	1.08040	05002	1.05194	27
3.3 3.4	85000	1.16800	.88080	1.12705	-01847	1.08876	.95118	1.05133	20
3.5	.85000	1.10741	.88732	1.12000	.01001	1.08813	-95173	1.05072	25
36	.85710	1.16072	.8878.4	1.12033	-01055	1.08740	-95229	1.05010	2.1
37	.85701	1.10003	.888,36	1.12507	.02008	1.08080	05284	1.04040	23
38	.85811	1.10535	.88888	1.12501	.02002	1.08622	-05340 -05305	1.04827	21
30	.85862	1.16466	.880.10	1.12435	.02110	1.08490	95451	1.0.1700	20
40	.85912	1.16398				1.08432	.05506	1.0.1705	10
.4 T	.85063	1.16320	80045	1.12303	.02224	1.08300	.05502	1.04044	18
42	.86014	1.10201	.80207	1.12230	.02331	1.08300	05018	1.04583	17
4.3	.86115	1.16124	.80201	1.12100	-02385	1.08243	-05073	1.04522	10
4.5	.86166	1.10050	80253	1.12041	.92439	1.08170	-05720	1.04461	15
46	.86216	1.15087	.80306	1.11075	.02493	0.180.1	05785	1.04401	1.4
47	.86267	1.15010	.80358	1.11000	-02547	1.08053	05841	1.0.13.10	13
48	.86318	1.15851	80410	1.11844	.02001	1.07000	-05052	1.04270	11
40	86368	1.15783	.80.103	1.11778	.02655 .02700	1.07804	.95952	1.04158	10
50	.86419	1.15715	.80515	1.11713	11		11	1.0.1007	0
51	.86470	1.15647	80507	1.116.18	.02763	1.07801	4000a	1.04030	8
52	.86521	1.15570	S0020 S0072	1.11582	.02872	1.07076	.00170	1.03070	7
53	.86572	1.15511	80725	1.11452	.02026	1.07013	-00232	1.03015	6
54 55	.86074	1.15375	80777	1.11387	.02080	1.07550	.00288	1.03855	5
56	.86725	1.15308	.80830	1.11321	.03034	1.07487	90344	1.03794	4
57	.86776	1.15240	.89883	1.11250	.0,3088	1.07425	.00,100	1.03734	3
58	.86827	1.15172	.80035	1.11101	-03143	1.07302	-00457	1.03074	2
50	.86878	1.15104	.80088	1.11120	-03107	1.07200	.96513	1.03513	1
(io	.86929	1.15037	.000.10	1.11061	.03252	1.07237	.00500	- 1.03.333	
,	CO-TAN	TAN.	CO-TAN	TAN.	CO-TAN	TAN.	CO-TAN	l TAN.	
	COLIAN	190 1 7.5	1	180	1	47°	11	46°)
	1 3	10		~~					

	1 4	4 °	1	11	4	40	1	1	1 4	<u> 14</u> 0	(
′	TAN.	Co-TAN.	'	′	TAN.	Co-tan.	′	'	TAN.	Co-tan.	/
0	.96569	1.03553	60	21	.97756	1.02295	30	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	. 99016	1.00994	17
3	.96738	1.03372	57	24	-97927	1.02117	36	44	-99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	-99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	1.4
	.96907	1.03192	54	27	.98098	1.01939	33	47	-99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	-99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	-99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	20	51	.99478	1.00525	0
II	.07180	1.02802	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.07246	1.02832	48	33	.08441	1.01583	27	53	-99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	-97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	-97472	1.02593	44	37	98671	1.01347	23	57	.99826	1.00175	3
17	-97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	42
18	.97586	1.02474	42	39	.98786	1.01229	21	59	-99942	1.00058	ĭ
19	.97643	1.02414	41	40	.98843	1.01170	20	60	I	r	0
20	.97700	1.02355	40								
,	CO-TAN.	TAN.	1	1	CO-TAN.	TAN.	1	,	CO-TAN.	TAN.	,
	45°			4.	5°			43	50		

6. Complete the following table using the table of sines cosines, etc.

From the above table what do you find about the sin 30° and the $\cos 60^{\circ}$, [that is, the \cos of $(90^{\circ} - 30^{\circ})$], also about the $\sin 20^{\circ}$ 10' and the \cos of $(90^{\circ} - 20^{\circ}$ 10'), that is, the $\cos 69^{\circ}$ 50'; about the $\tan 12^{\circ}$ and the \cot of $(90^{\circ} - 12^{\circ})$ or $\cot 78^{\circ}$. In other words, this is true.

The sine of any angle less than 90° equals the co-function or cosine of (90° minus that angle), also, the tangent of any angle less than 90°, for example, the tangent of 12° equals the co-function or cotangent of $(90^{\circ} - 12^{\circ})$

or equals the cot of 78°. In the same way the sine 20° $10' = \cos .69^{\circ} 50'$, the cot $48^{\circ} = \tan (90^{\circ} - 48^{\circ})$ or $\tan 42^{\circ}$ the $\cos 30^{\circ} = \sin . (90^{\circ} - 30^{\circ})$ or $\sin 60^{\circ}$, etc. This is a good rule to remember.

62. Line Values of Functions.—The values of the sine.

cosine, tangent and cotangent of angles can be represented by the lengths of lines as follows: In Fig. 109 the circle with center O is drawn with a radius (one-half the diameter) equal to 1 in. CD is a horizontal and EF a vertical diameter. The radius OA makes any convenient angle. "a" with radius OC and from point A line

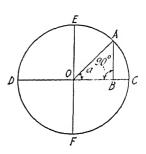


Fig. 109.

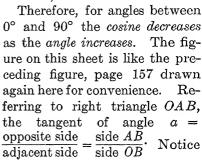
AB is drawn at right angles to line OC making the angle at B equal to 90° and giving the right triangle OAB. From the definition of the sine $\left(= \frac{\text{opposite side}}{\text{hypothenuse}} \right)$ have in the right triangle OAB. The sine of angle side AB. But OA = 1 by construction (since we made the circle with OA = 1 in.). Therefore sine $a = \operatorname{side} AB$ or side AB. Hence the sine of a is equal to the length of the line AB. In the figure, AB measures: .5 of an inch. Therefore sine a = .5. Notice that if angle a were smaller the line AB would be smaller, that is, sine a would be smaller. If angle a were larger, line AB would be larger and therefore sine a would be larger. When angle $a = 0^{\circ}$, line AB = 0 in, and sine a = 0, that is, sine $0^{\circ} = 0$. When $a = 90^{\circ}$, line AB = lineOE in length or 1 in, and therefore sine 90° = 1. There

fore for angle between 0° and 90° the sine *increases* when the angle *increases*.

Referring again to Fig. 109, in the right triangle OAB the cosine of angle $a = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{\text{side }OB}{\text{hyp. }OA} = \frac{\text{side }OB}{1}$ or side OB.

That is, the line OB equals to scale the cosine of angle a. When angle a = 0, OB = OC, and $\cos C$

a = 1. When angle $a = 90^{\circ}$, line OB = 0, and cosine $90^{\circ} = 0$.



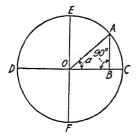


Fig. 110.

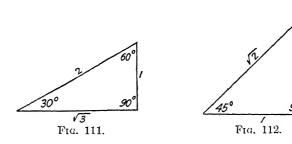
that as angle a increases, side AC (that is, the numerator of the fraction which is the tangent) increases, and at the same time side OB, the denominator of the fraction, decreases. Therefore as angle a increases the tangent increases and also if angle a decreases the tangent of angle a decreases.

In right triangle OAB the cotangent of angle $a=\frac{\text{adjacent side}}{\text{opposite side}} = \frac{\text{side }OB}{\text{side }AB}$. Then as angle a increases, side OB decreases and side AB increases; therefore the fraction representing the cotangent decreases. As angle a decreases side OB increases and side AB decreases, and the value of the fraction increases, that is, the cotangent

increases. The following table for values of 0° and 90° angles should be carefully studied and understood from Fig. 110.

$_{ m Angle}$	Sine	Cosine	Tangent	Cotangent
0°	0	1	0	Infinity
90°	1	0	Infinity	0

63. Values of the Sine, Cosine, Tangent, and Cotangent for the Common Angles.—In a 30°-60° right



triangle shown in Fig. 111 the sides always have a ratio of 1, 2, and $\sqrt{3}$ as given. You can prove this for yourselves by carefully constructing such a triangle on the drawing board and measuring the lengths of the sides. Therefore from our definition of the sine, cosine, etc., of an angle we have

Sin
$$30^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{1}{2} = .5$$

Cos $30^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{\sqrt{3}}{2} = .866$
Tan $30^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{1}{\sqrt{3}} = .577$

Cot
$$30^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{\sqrt{3}}{1} = 1.732$$

Sin $60^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{\sqrt{3}}{2} = .866$
Cos $60^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{1}{2} = .5$
Tan $60^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{\sqrt{3}}{1} = 1.732$
Cot $60^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{1}{1/2} = .577$

In a 45°-45° right triangle as shown in Fig. 112 the sides always have a ratio of 1, 1 and $\sqrt{2}$ as shown. Therefore for this figure:

Sin
$$45^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = .707$$

Cos $45^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = .707$
Tan $45^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{1}{1} = 1.00$
Cot $45^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{1}{1} = 1.00$

These values are tabulated below for reference.

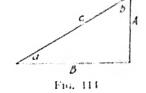
Angle	Sine	Cosine	Tangent	Cotangent
30°	. 5	.866	. 577	1.732
45°	.707	.707	1.00	1.00
60°	.866	.5	1.732	. 577

64. Methods of Working out Right Triangles.—In working out problems by the use of the sine, cosine, etc., we use one or the other, according to the sides of the triangle which are to be found and the sides which are

or example in Fig. 113 the hyp. ~ 10 in, and acute angle $\approx 30^{\circ}$. We want to find side and angle b. In any right triangle the sum of ate angles equals 90° , therefore angle $b = 10^{\circ} = 10^{\circ}$ and angle $b = 10^{\circ} = 10^{\circ}$ or 10° . To find ave sine $10^{\circ} = 10^{\circ}$ opp. side 10° in Fine

 $\frac{4}{3}$, side $A = 10 \times \sin 30^{\circ}$ (sine $30^{\circ} = 5$.





. 113,

Le $A = 10 \times .5$ or 5 in. To find side B osine as follows:

 $\cos 30^{\circ}$ adj. side side B hyp. 10

He $B = 10 \times \cos(30^\circ)$, (cos. 30° = .866), • $B = 10 \times .866$ or 8.66 in,

explanation just given and using Fig. 114. following table. The first set of value orked out as a guide.

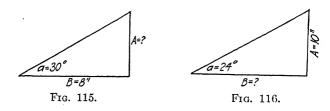
e a	Angle 5	ple to	o to	isto di
. ,	60 '	16	8	1., 9
4.0		21		
	30"	38		
**		7.5		
•		62		
	751	35		

^{* *** 3}

Draw a separate triangle for each set of values and put the lengths of sides and sizes of angles on each figure as well as putting them in the table.

If as in the triangle shown (Fig. 115) we had given angle $a = 30^{\circ}$ and side B = 8, to find side A we use the tanget as follows: $\tan 30^{\circ} = \frac{\text{side } A}{\text{side } B}$, therefore side $A = \text{side } B \times \tan 30^{\circ}$ or side $A = 8 \times .577 = 4.62$.

If as in the triangle shown in Fig. 116 we have the side



A = 10 and angle $a = 24^{\circ}$ and wish to find side B we use the cotangent as follows:

$$\cot 24^{\circ} = \frac{\text{side } B}{10}$$

Therefore side $B = 10 \times \text{cot } 24^{\circ}$ or side $B = 10 \times 2.25$ or 22.5. From the explanation just given and using Fig. 117 complete the following table.

	Angle a	Angle b	Side A	Side B
$_{2}^{1}$	000	29°	16	
3	80°	36°	35	8.4
4 5	15°	75°	7.5	45
6	60°	.0	7.0	24

When the base B and angle a are given (Fig. 118) to find the hypothenuse proceed as follows:

$$\cos 40^{\circ} = \frac{\text{side } B}{\text{hyp.}}$$
$$\cos 40^{\circ} = \frac{20}{\text{hyp.}}$$

Fig. 117.

or

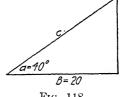
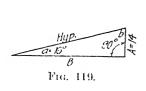


Fig. 118.

Therefore $\cos 40^{\circ} \times \text{hyp.} = 20$ and from this the hyp. =

$$\frac{20}{\cos 40^{\circ}} = \frac{20}{.766} = 26.1$$

When the side A and angle a are given (Fig. 119) to find the hypothenuse proceed as follows:



$$\sin 16^{\circ} = \frac{\text{side } A}{\text{hyp.}} \text{ or } \sin 16^{\circ} = \frac{14}{\text{hyp.}}$$

Therefore hyp. $\times \sin 16^{\circ} = 14$ and hyp. $= \frac{14}{\sin 16^{\circ}}$ hyp. = $\frac{14}{276}$ = 50.7

	Angle a	Angle b	Side A	Side B
1	12°		10	
2		16° 30′		20
3	43°		15	
4		27° 10′		63
5	36°		. 23.8	
6		32°		42

7. From the explanation just given fill in the following tax

When we have two sides of a right triangle and to find the angles we proceed as follows: In the tr shown in Fig. 120 A = 12, B = 16, hyp. = 20 find angles a and b.

Sin
$$a = \frac{\text{opp. side}}{\text{hyp.}} = \frac{12}{20} = \frac{3}{5} = .6$$
. If sin $a = .6$

angle a is one whose sine is .6. We therefore look tables for an angle whose sine is .6 and on page 14 the number .59995 which is the nearest one to .6 the angle is therefore 36° 52′.

We can find angle a also from the rule

$$\cos a = \frac{\text{adj. side}}{\text{hyp.}} = \frac{16}{20} = \frac{8}{10} = .8$$

Looking in the tables for the angle whose cosine is . we find on page 142 the angle 36° 52'.

We can find angle a also from the tangent r follows:

$$Tan \alpha = \frac{opp. \ side}{adi. \ side} = \frac{12}{16} = \frac{3}{4} = .75$$

and therefore (page 154) angle $a=36^{\circ}$ 52', or fro cotangent rule

$$\cot a = \frac{16}{12} = \frac{4}{3} = 1.33$$

and from the tables (page 154) angle $a = 36^{\circ} 52'$.

Having obtained angle a from any one of the rules just given, angle b equals 90° – angle a, since angle a + angle $b = 90^{\circ}$.

8. From the explanation just given and using Fig. 121, find the value of the sine of angle a and the value of angle a, completing the following table.

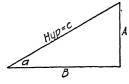


Fig. 121.

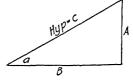


Fig. 122.

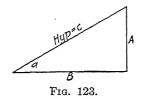
	Side A	Нур.	Sine a	Angle a
1 2	25 7	50 35		
3	28	38		
4 5	15 12	$\frac{20}{32}$		

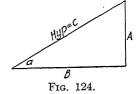
9. Using Fig. 122, find the value of the cosine of angle a and angle a, completing the following table.

	Side B	Нур.	Cosine a	Angle a
1 2 3 4 5	10 3 30 16 14	20 4 75 28 32		The second secon

10. For Fig. 123, find the tangent of angle a and the value of angle a, completing the following table.

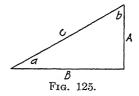
	Side A	Side B	Tangent a	Angle a
1 2 3 4 5	10 36 3 56 38	24 70 8 40 112		





11. For Fig. 124, find the cotangent of angle a and the value of angle a, completing the following table.

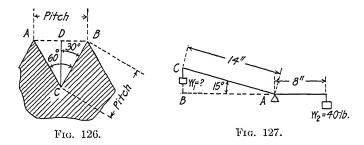
	Side A	Side B	Cotangent a	Angle a
1 2 3 4 5	7 27 14 12 23	32 38 18 16 42		



12. Using Fig. 125, complete the following table, drawing a separate triangle for each problem.

	Angle α	Angle b	Side A	Side B	Side C
$1\\2\\3\\4$	60°	40° 32°	15 20	35	75
5 6	15°		25	30	50

- 65. Applications of Triangular Functions.—Following are applications of the preceding rules to the solution of problems in daily shop work.
- 13. To find the depth of a V thread. Fig. 126 shows a section of the thread. To find the depth of the thread or distance CD



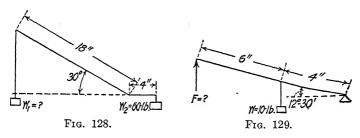
in triangle *DBC*. Side $CD = \text{side } CB \times \cos 30^{\circ} = \text{pitch } \times .866$. The pitch in in. always equals $\frac{1}{\text{threads per in.}}$

Find the depth of V threads as follows: 8 per in., 20 per in., 12 per in., and 40 per in.

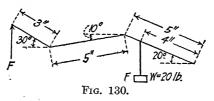
- 14. In a right triangle the hypothenuse is 20 in. and the height is to be twice the base. Find the height, base, and all the angles of the triangle. (Note.—Let the base be x in. and the height 2x in. The tangent of the base angle will then be $\frac{2x}{x}$ or 2.)
 - 15. Fig. 127 shows a first-class lever. If $W_2 = 40$ lb. to find

 W_1 we have the rule $40 \times 8 = W_1 \times \text{distance } AB$, but length $AB = 14 \times \cos 15^\circ = 14 \times .966 = 13.5$, therefore $40 \times 8 = W_1 \times 13.5$ and $W_1 = \frac{40 \times 8}{13.5} = 23.7 \text{ lb.}$ Ans.

16. With a first-class lever as in Fig. 128, find W_1 necessary to balance W_2 with lever arms as shown.

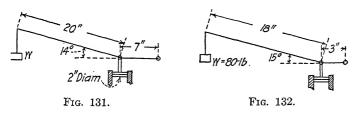


17. As per Fig. 129, find the force necessary at F to balance the weight of 10 lb. shown.

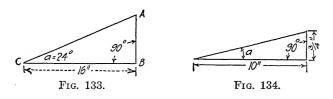


- 18. As shown in Fig. 130, find the force necessary at F to balance the 20 lb. What class of lever is this one?
- 19. What weight in lb. at W (Fig. 131) is required to set the safety valve so as to lift at 200 lb. per sq. in. pressure with a 2-in. diam. valve?
- 20. As per Fig. 132, what size valve is required with steam at 150 lb. per sq. in. to lift the weight of 80 lb. at W, with the lever arms as shown?
- 21. In a laying out job the distance CB Fig. 133 is taken as 16 in., the angle $a=24^{\circ}$. Find the distance between centers A and B.
- 22. The tangent of an angle is .45573. What are its sinc, cotangent, and cosine?

- 23. For the wedge shown (Fig. 134) find the angle between its sides.
- 24. Find the angle between the sides of a wedge 1 ft. long, in. wide at one end and 1/4 in. wide at the other end.
- 25. The sides of a wrought-iron wedge form an angle of 2° 23'. Find the taper (decrease in thickness) per ft. of length.



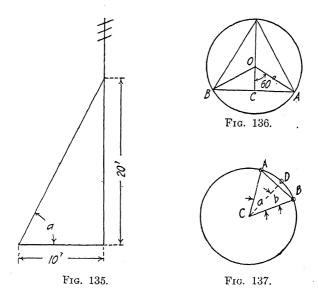
- 26. A reamer $1\frac{5}{6}$ in. at the small end is 16 in. long and has a aper of $\frac{3}{4}$ in. in. 12 in. ($\frac{3}{4}$ in. increase in diam. per ft.). What is he diameter at the large end and what is the angle between the wo sides?
- 27. Find the angle which the sides of a wedge make with one nother for a taper of $\frac{7}{8}$ in. per ft.



- 28. Fig. 135 shows a telegraph pole with a guy wire attached 1 ft. up on pole and anchored 10 ft. from base of pole. Find ngth of wire required. Allow 2 ft. extra for fastening wire. Note.—Find first angle a, then get side C from the sine or cosine le.)
- 29. Find the angle of incline for a grade of 3 per cent., and the amber of feet rise in a distance of 2 miles. (A 1 per cent. grade one which rises 1 ft. vertically for every 100 ft. horizontally.)

How many foot-pounds of work are required to lift a 600-ton train up this distance against gravity?

- 30. Find the average angle of incline for the roadbed from Altoona, Pa., to Gallitzin, Pa., the rise being about 980 ft. in 10 miles.
- 31. Two adjacent spokes in a wheel make with one another an angle whose cosine is .91355. What is the angle? How many spokes are there in the wheel?
 - 32. A belt passes over two pulleys whose centers are 11 ft.



apart. The pulley diameters are 18 in. and 42 in. Find the angle between the two sides of the belt.

- 33. Find the angle between the sides of a belt connecting two pulleys 24 in. and 12 in. in diam. and with centers 6 ft. apart.
- 34. Find the length of a side of an equilateral triangle inscribed in a 24-in. diam. circle. Fig. 136 shows the circle and triangle, Drawing the radii OA and OB we have the triangle OAB. Line OC divides this triangle into two equal right triangles OAC and OCB.

ingle $AOC = 60^{\circ}$. To find side AB we find AC first, then $AB = \times AC$. Now in triangle AOC

 $\frac{AC}{OA} = \sin 60^{\circ}$ $AC = OA \times \sin 60^{\circ} = OA \times .866$

But OA is the radius of the circle or 12 in.

herefore

herefore $AC = 12 \times .866 = 10.4$ in. and $AB = 2 \times AC = 2 \times 0.4 = 20.8$ in. Ans.

35. Find the length of one side of each of the following equalded inscribed figures.

Number of equal sides

Diam of circle in which figure is inscribed

18 in.

7

24 in.

36. As shown in Fig. 137, find the distance AB to be laid off for rilling two of eight equally spaced holes on an 8-in. diam. circle.

angle
$$a = \frac{360^{\circ}}{8} = 45^{\circ}$$

angle $b = \frac{45}{2} = 22\frac{1}{2}^{\circ}$.

herefore in right triangle BCD

 $\frac{\text{side }BD}{\text{side }BC} = \sin \text{ angle } b, \text{ therefore}$ $\text{side }BD = \text{side }BC \times \sin b.$ $= 4 \text{ in. } \times \sin 22\frac{1}{2}^{\circ}.$ $= 4 \times .383 = 1.532 \text{ in.}$ and $AB = 2 \times BD = 2 \times 1.53 = 3.06 \text{ in.}$ Ans.

37. Find the distance at which to set dividers for laying off 12 qually spaced holes on a 42-in. diam. circle.

38. What is the distance in in. between any two consecutive enters when 17 are spaced an equal distance apart on a 1-ft. am. circle?

39. If the straight distance between two consecutive centers a 12-in. diam. circle is 1½ in., what is the angle formed at the enter of the circle by the radii drawn to these two centers?

40. In the sketch, Fig. 138, showing a cross-sectional view of a

boiler shell 72-in. diam. with water line 20 in. above the center of shell, find the length of the water line "x" in in.

- 41. What is the taper per ft. of length for a wedge, the sides of which form an angle whose tangent is .0626?
 - **42.** In Fig. 139, diameter AB = 2 in., chords AC and AD are

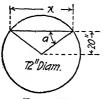


Fig. 138.

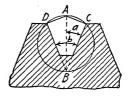
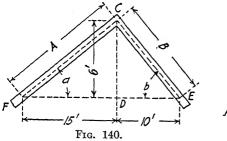


Fig. 139.

each $\frac{1}{2}$ in. and angles C and D are each 90°. Find angle a and angle b. Angle b is that used in constructing the "acme" thread. Remember how this figure is drawn, to get this angle for an "acme thread."

43. In Fig. 140, find the length of the center lines of rafters A





and B. (Note.—The tangent of angle $a = \frac{6}{15} = \frac{2}{5} = .4$. Find angle a in the tables. Then knowing a and the 15-ft. side of the triangle CDF find side A. In the same way to find side B, find angle b first, then work out the right triangle CDE for side B knowing angle b and the 10-ft. side.

44. A line shaft is 18 ft. above and 15 ft. to the left of a motor

shaft. Find the distance between these shafts by drawing the right triangle, using the tables to get one of the acute angles and then the sine or cosine rule to get the hypothenuse of the triangle or the answer.

45. Find the angle between the belt on two sides of pulleys 18 in. and 42 in. in diam. and centers 11 ft. apart. Make a sketch of the belt and pulleys.

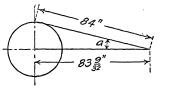


Fig. 142.



Fig. 143.

- 46. Find the length of plank required to reach from A to B, as per Fig. 141.
- 47. In Fig. 142 with crank pin on the top quarter the distance from center of driver to center of cross-head is $83\%_2$ in. The main rod is 84 in. long. Find the angle a and the stroke of the engine.

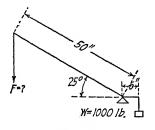


Fig. 144.

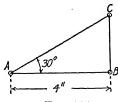
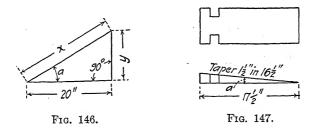


Fig. 145.

- 48. As in Fig. 143 find to what distance you would set dividers for laying off 26 equally spaced stud holes on a 23½-in. diam. circle on a cylinder head.
- 49. As shown in Fig. 144, what force at F is necessary to lift the weight of 1000 lb.?

- **50.** Three holes are to be laid out as in Fig. 145. Find distances BC and AC.
 - **51.** In Fig. 146, $\cos a = .955$. Find angle a and sides x and y.
- **52.** The sides of a wrought-iron wedge form an angle of 2° 23′. What is the taper per ft. of length?
- 53. Find the taper per ft. of length of a wedge the sides of which form an angle whose tangent is .0625.



- 54. The shank on a drill socket has a taper of $\frac{5}{6}$ in. in 12 in., is $\frac{5}{6}$ in. in diam. at the small end and 5 in. long. What is its diameter at the large end (to the nearest 64th of an in.) and the angle between the sides?
- 55. If a piece of work is to be planed to a taper of $\frac{1}{16}$ in. in 1 in. for a length of $\frac{22}{16}$ in., how high would you block up the thin end?

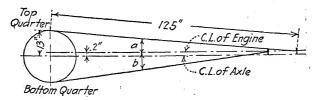


Fig. 148.

- 56. To what distance would you set dividers for laying off 4 equally spaced centers on an 8-in. diam. circle?
- 57. In Fig. 147 showing two views of a wedge find angle a in degrees and minutes. How high would you block up the thin end for planing the tapered surface?

58. By what distance in in. is the center of the cross-head (Fig. 148) drawn back of the middle of its stroke when the crank pin is on both the top and bottom quarters?

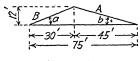


Fig. 149.

59. In Fig. 149, find the value of angles a and b and sides A and B.

60. For Fig. 150, find the distance between the centers A and B.

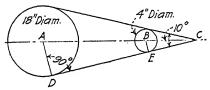
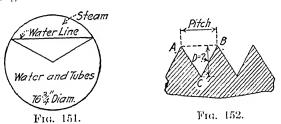


Fig. 150.

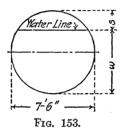
(Note.—Find first the hypothenuse of each of the right triangles ADC and BEC.)

61. Fig. 151 shows a cross-sectional view of a boiler shell 76¾



in. in diam, and the water line is $21\frac{1}{2}$ in, above the center line of the boiler. Find (1) the area of the steam segment, (2) the area of the water segment, and (3) the net water area when there are 465 2-in, outside diam. (O. D.) tubes.

- 62. Find the depth of a V thread in terms of the number of threads per in. Triangle ABC, Fig. 152, is equiangular and therefore equilateral.
- 63. Fig. 153 represents a cross-sectional view of a boiler shell 7 ft. 6 in. in diam. and has 482 15%-in. O. D. tubes, the ratio of the height of the steam segment to the height of the water



segment or S to W is as 2 is to 7, thus $\frac{S}{W} = \frac{2}{7}$. Find (1) the height and area of the steam segment; (2) the height and area of the water segment; (3) the net area of the water segment after deducting area of the tubes.

64. Determine the distance in in. between the center of the

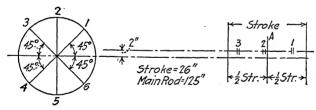


Fig. 154.

cross-head and the center of the stroke or point "A," Fig. 154, when the crank pin occupies the six different positions marked 1-2-3-4-5 and 6. The center line of the cylinder is 2 in. above the center line of the axle. How would these distances be changed, first by shortening the main rod, and second by lengthening the main rod?

CHAPTER XVIII

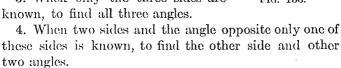
THE MEASUREMENT OF OBLIQUE TRIANGLES

66. Methods of Working out Oblique Triangles.—In the following problems we are to work out triangles which have no right angles, that is oblique triangles.

How we work out these oblique triangles depends on which sides and angles and how many of either are given, and which sides and angles we wish to find.

In any problem we will have one of the four following cases.

- 1. When one side and two angles are known, to find the remaining two sides and angle.
- 2. When two sides and their included angle are known, to find the other side and other two angles.
- 3. When only the three sides are



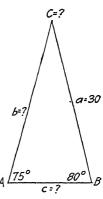


Fig. 155.

CASE I

Example.—Given, any one side and any two angles of the triangle. To find the other two sides and angle.

In Fig. 155 suppose we have given angle $A = 75^{\circ}$, 177

12

angle $B = 80^{\circ}$ and side a = 30. To find angle C and sides b and c we proceed as follows:

In every triangle the three angles added together equal 180°. Therefore angle A + angle B + angle C = 180°. But angle A = 75° and angle B = 80°. Therefore 75° + 80° + angle C = 180° and angle C = 180° - 75° - 80° or 25°.

To find the sides b and c we have the rule. In any triangle any two sides have the same ratio as the *sines* of their opposite angles. Therefore the figure shows

$$\frac{\text{side } b}{\text{side } a} = \frac{\text{sine angle } B}{\text{sine angle } A}$$

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine angle } C}{\text{sine angle } A}$$

also

using the last rule we have

$$\frac{\text{side } c}{30} = \frac{\text{sine } 25^{\circ}}{\text{sine } 75^{\circ}} \text{ or } \frac{\text{side } c}{30} = \frac{.423}{.966}$$

from which side $c = \frac{30 \times .423}{.966}$ or 13.14 in.

also from the rule that

$$\frac{\text{side } b}{\text{side } a} = \frac{\text{sine angle } B}{\text{sine angle } A}$$

$$\frac{\text{side } b}{30} = \frac{\text{sine } 80^{\circ}}{\text{sine } 75^{\circ}}$$

$$\frac{\text{side } b}{30} = \frac{.985}{.966}$$

From which

and side $b = \frac{30 \times .985}{.966}$ or 30.6 in.

Example.—In Fig. 156 angle $A = 56^{\circ}$ 10' angle $B = 64^{\circ}$ 30' and side b = 21. We want to find angle C and sides a and c.

Angle
$$C = 180^{\circ} - (\text{angle } A + \text{angle } B)$$

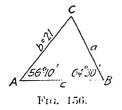
 $= 180^{\circ} - (56^{\circ} 10' + 64^{\circ} 30')$
 $= 59^{\circ} 20'$
side $a = \sin A$
side $b = \sin B$
side $a = .830$
 $21 = .903$
side $a = \frac{21 \times .830}{.903}$ or 19.3

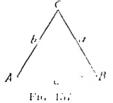
Therefore

To find side c, we have the rule:

$$\frac{\text{side }c}{\text{side }b} = \frac{\sin C}{\sin B} \text{ therefore } \frac{\text{side }c}{21} \qquad .860$$

from which, side $c = \frac{21 \times .860}{.903}$ or 20.0





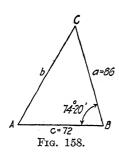
1. From the explanation just given and using Lig. 157, complete the following table, drawing a separate triangle for each problem. The answers are given to the first two problems to help in under standing them.

	Angle A	Angle B	$\lambda_{tiple}(e)$, sele es	οĎ	, 4
1	60"	65" 20'	[54 ' 40']	18 m	Hannet	In the
1 2 3	80"	30 1307	69 30	[16 Gin]	1159m t	11 1.
3	701		10 501			700
4		48	72		4 2 11.	
5 6	37"	48 87 451		1846		
6		42 167	85			
7		30	60	Sec. 11.		

CASE II

Example.—Given, any two sides and their included angle. To find the remaining side and remaining angles.

In Fig. 158 we have given side a = 86, side c = 72,



and angle
$$B = 74^{\circ} 20'$$
. In order to find side b and angles A and C we use the following rules.

$$\frac{\tan \frac{1}{2}(\text{angle } A - \text{angle } C)}{\tan \frac{1}{2}(\text{angle } A + \text{angle } C)} = \frac{\text{side } a - \text{side } c}{\text{side } a + \text{side } c}$$

Now angle A + angle B + angle $C = 180^{\circ}$

Therefore Angle A+ Angle $C=180^{\circ}-$ Angle B or $180^{\circ}-74^{\circ}$ 20' or 105° 40'.

Then using the rule above, we have

$$\frac{\tan \frac{1}{2} (\text{angle } A - \text{angle } C)}{\tan \frac{1}{2} (105^{\circ}40')} = \frac{86 - 72}{86 + 72}$$

or tan ½(angle A – angle C) = tan ½(105° 40′) $\times \frac{14}{158}$.

The tan $\frac{1}{2}(105^{\circ} 40') \times \frac{14}{158} = \tan 52^{\circ} 50' \times \frac{14}{158}$ or $\frac{1.319 \times 14}{158}$. This fraction worked out equals .117.

If $\tan \frac{1}{2}(\text{angle }A - \text{angle }C) = .117$ we find from the table of tangents that $\frac{1}{2}(\text{angle }A - \text{angle }C) = 6^{\circ}$ 40' and angle $A - \text{angle }C = 2 \times [6^{\circ}$ 40'] or 13° 20', but angle $A + \text{angle }C = 105^{\circ}$ 40'. Adding, 2 × angle $A = 119^{\circ}$ and angle $A = 59^{\circ}$ 30'.

Angle
$$C = 180^{\circ}$$
 — angle A — angle B or 180° — 59° $30'$ — 74° $20'$ or 46° $10'$ To find side b we have as in Case I.

$$\frac{\text{side } b}{\text{side } a} = \frac{\sin B}{\sin A}$$

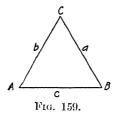
That is

$$\frac{\text{side } b}{86} = \frac{\sin 74^{\circ} 20'}{\sin 59^{\circ} 30'}$$

$$\text{side } b = \frac{86 \times .963}{.862} = 96.1$$

Therefore

2. From the explanation just given and using Fig. 159, complete the following table, drawing a separate triangle for each problem.



The answers in parenthesis() are given for the first two problem to help in understanding them.

	Angle A	Angle B	Angle C	Side a	Side b	Side c
1	(116° 10′)	40° 20′	(23° 30′)	72	(51.9)	32
2	58° 40′	(70°)	(51° 20')	(5.90)	65	5.4
3	}		70°	120	110	
4	36° 40′				98	103
5		86°		18		24
6			98° 20′	7.92	12.3	
7	88° 40′				2.35	4.25
8	1	23"		56		63

CASE III

Example.—Given, the three sides of a triangle. To find the three angles. In Fig. 160 we have given the sides as follows: side a=4, side b=6, side c=8. To find angles A, B and C we use the following rules in which $s=\frac{\text{side }a+\text{side }b+\text{side }c}{2}$ that is one-half the sum of the three sides of the triangle.





(1)
$$\cos \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$
 (2) $\cos \frac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}}$ (3) $\cos \frac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}}$

In Fig. 160 we have $s = \frac{\text{side } a + \text{side } b + \text{side } c}{2} = \frac{4+6+8}{2}$ or 9, and using rule (1) above:

 $\cos \frac{1}{2}A = \sqrt{\frac{9(9-4)}{6\times8}} \text{ which equals } \sqrt{\frac{9\times5}{48}} \text{ or .9683.}$

If $\cos \frac{1}{2}A = .9683$ from the table of cosines we find $\frac{1}{2}A = 14^{\circ} 28'$, and therefore $A = 28^{\circ} 56'$.

By rule (2) above, $\cos \frac{1}{2}B = \sqrt{\frac{9(9-6)}{4\times8}}$ which equals $\sqrt{\frac{27}{32}}$ or .9185, and from the tables $\frac{1}{2}B = 23^{\circ}$ 17′. Therefore $B = 46^{\circ}$ 34′. Then angle $C = 180^{\circ}$ – (angle $A + 180^{\circ}$)

angle B) or $180^{\circ} - (28^{\circ} 56' + 46^{\circ} 34')$ which equals 104° 30′.

3. Using Fig. 161 and from the explanation just given complete the following table, drawing a separate triangle for each problem. The answers to the first problem are already given.

	Side a	Side b	Side c	Angle A	Angle B	Angle C
		Side 0	Side 8	Angle A	Tingle 2	1
1	3	5	6	(29° 56′)	(56° 16′)	(93° 48′)
2	4	6	5			
3	12	10	9			
4	5.6	- 4.8	3.7			
5	5.8	6.5	8.4			
6	60	80	48			
7	70	62	39			
8	56	43	36			

Put on each triangle the lengths of the sides given and the values of the angles found.

CASE IV

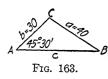
Example.—Given any two sides of the triangle and the angle opposite one of them To find the remaining side and other two angles. Suppose, as in Fig. 162, we have given side a = 24, side b = 30.5and angle $A = 31^{\circ} 20'$. To find angle B we have by Case I:

Fig. 162.

$$\frac{\sin B}{\sin A} = \frac{\text{side } b}{\text{side } a} \text{ or } \sin B = \frac{30.5 \times .520}{24} \text{ or } .6608$$

and from the table of sines we find angle $B = 41^{\circ} 22'$. But an angle and its *supplement* have the same sine, the supplement of an angle being that angle which added to the given angle will equal 180 degrees. For example, suppose we have given an angle of $38^{\circ} 40'$, then $180^{\circ} - 38^{\circ} 40' = 141^{\circ} 20'$ and $141^{\circ} 20'$ is the supplement of the angle $38^{\circ} 40'$.

It is possible, therefore, to have two values for angle B in this problem. Hence another value for angle B in this problem is $180^{\circ} - 41^{\circ} 22'$ or $138^{\circ} 38'$. Call this value B_1 ; and since we have two values for angle B, we must have two triangles, each having an angle $A = 31^{\circ} 20'$, a side a = 24, and a side b = 30.5, and hence two



values for angle C and side c. Call these values C and C_1 . Then $C = 180^{\circ} - (A + B)$ or $180^{\circ} - (31^{\circ} \ 20' + 41^{\circ} \ 22')$ which equals $107^{\circ} \ 18'$; also $C_1 = 180^{\circ} - (A + B_1)$ or $180^{\circ} - (31^{\circ} \ 20' + 138^{\circ} \ 38')$ or $10^{\circ} \ 2'$.

We have now to find side c and side c_1 . By Case I we have

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A} \text{ and } \frac{\text{side } c_1}{\text{side } a} = \frac{\text{sine } C_1}{\text{sine } A}$$
Therefore side $c = \frac{24 \times .95476}{.520} \text{ or } 44.06$
and side $c_1 = \frac{24 \times .17422}{.520} \text{ or } 8.04$

We will not always have two solutions, that is, two values for the unknown angles and sides. For instance, suppose as in Fig. 163 we had given side a=40, side b=30 and angle $A=45^{\circ}30'$. Then by Case I we have

$$\frac{\text{sine } B}{\text{sine } A} = \frac{\text{side } b}{\text{side } a}$$
 or sine $B = \frac{30 \times .71325}{40}$ or .5349

and from the table of sines we have $B = 32^{\circ} 20'$.

The supplement of angle B is $180^{\circ} - 32^{\circ} 20'$ or $147^{\circ} 40'$, that is, angle B_1 . But the angles $A + B_1 = 45^{\circ} 30'$

+ 147° 40′ or 193° 10′ which is impossible, since the sum of any two angles of a triangle must be less than 180°, hence we have only one solution to the problem: Angle $C = 180^{\circ} - (A + B)$ or $180^{\circ} - (45^{\circ} \ 30' + 32^{\circ})$ 20') or 102° 10', since

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A}$$
, side $c = \frac{40 \times .97754}{.71325}$ or 54.8

Again, suppose as shown in Fig. 164 we had given side a = 32, side b = 44, and angle $B = 112^{\circ} 30'$. Then by Case I we have $\frac{\sin A}{\sin B} = \frac{\text{side } a}{\text{side } b}$

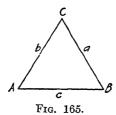
Now sine $112^{\circ} 30' = \text{sine } (180^{\circ} - 180)^{\circ}$ 112° 30') or sine 67° 30' (since the sine of any angle less than 180°. equals the sine of 180° minus that an-Therefore sine $A = \frac{32 \times .92388}{44}$ or .67191, and from the table A =42° 13′. Angle $C = 180^{\circ} - (A + B) A$ or $180^{\circ} - (42^{\circ} 13' + 112^{\circ} 30')$ or 25° Fig. 164. 17'; also since $\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A}$, then side $c = \frac{32 \times .42709}{.67191}$ or 20.34.

It is evident, since the given angle is greater than 90°, that there is only one solution to this problem.

We now have the following rules by which we may tell at a glance how many solutions there are to a problem under this case (Case IV).

- 1. If the side opposite the given angle is less than the other given side there are two solutions.
- 2. If the side opposite the given angle is greater than the other given side there is only one solution.

- 3. If the given angle is greater than 90° there is only one solution.
- 4. From the explanation just given and using Fig. 165 as a guide complete the following table, drawing a separate triangle for each problem. The answers to the first problem are already worked out.

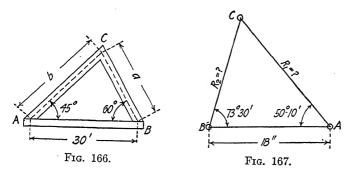


Angle A Angle BAngle C Side a Side b Side c 32° 10′ $B = 45^{\circ}13'$ c = 111 $C = 102^{\circ}37'$ 8 6 $B_1 = 134°47' | C_1 = 13°3'$ $c_1 = 2.54$ 26° 40′ 2 10 3 102° 50' 73 65 4 39° 10′ 28 33 5 59° 16 14 6 120° 50′ 48 42 7 83° 20' 47 51 128° 30′ 38 30

- 67. Applications of Triangular Functions to Oblique Triangles.—Following are applications to the solution of practical problems of the preceding rules for oblique triangles.
- **5.** In Fig. 166 is shown the center lines of a roof truss. Find the angle C and the length of the rafters a and b.
- **6.** In Fig. 167 find the radii R_1 and R_2 for laying out the three centers A, B, and C so that they may be located as shown.

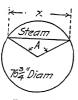
7. In Fig. 168 the diameter of a boiler is given as 76¾ in. and angle "A" is 150°. Find the length of water line "x."

8. It is desired to find the distance between two points A and B on opposite sides of a river. If the distance from B to a point C, Fig. 169, is found to be 128 ft. and the angles ABC and BCA are



found to be 45° 40′ and 50° 20′ respectively, what is the distance AB?

9. It is desired to find the height of the stack shown in Fig. 170. The angle of elevation of a point in the horizontal plane





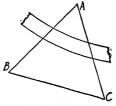


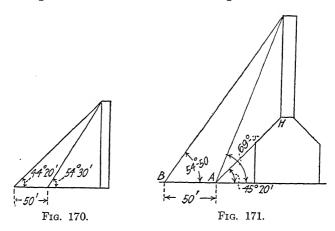
Fig. 169.

of the base of the stack is 54° 30′, and the angle of elevation at a point 50 ft. farther away is 44° 20′. Find the height of the stack.

10. The angle of elevation of the top of a stack at a point A, as shown in Fig. 171, is 69° 20′ and the angle of elevation at a point B, 50 ft. farther away, is 54° 50′. If the angle of elevation at

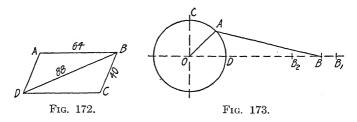
point A of the top of the building is 45° 20', find height of stack and height of building.

11. Two trains start from the same station at the same time traveling on tracks which intersect at an angle of 64° 20′. If the



trains travel at the rate of 35 and 50 m.p.h., respectively, how far apart are they at the end of 45 min.?

12. A 1/4-in. steel plate has the form of a parallelogram as shown in Fig. 172. If the sides are 64 in. and 40 in. in length and the

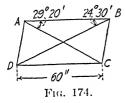


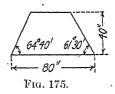
longer diagonal is 88 in. in length, find the angles A, B, C, and D, the area of the plate, its volume, and also its weight if 1 cu. in. of the metal weighs .283 lb.

13. In a steam engine the crank OA (Fig. 173) is 13 in. long, and

the connecting rod AB is 125 in. long. How far has the center B of the cross-head moved from its head end dead center position B_1 when the crank pin is midway between C and D?

14. Given a ½-in, steel plate with the shape as shown in Fig. 174. Find its area in sq. in, and its weight if steel weighs .28 lb, per cu. in.





15. Given a 3%-in, steel plate as shown in Fig. 175 with its longest side 80 in, and width 40 in. Find the non-parallel sides, its area in sq. in., and its weight.

CHAPTER XIX

ELECTRICITY

68. Definitions and Calculation of Resistance.—In making electrical calculations we need to understand the meaning of electrical "resistance" and how to calculate Every wire or electric circuit or portion of a circuit which conducts an electric current offers a "resistance" to the flow of the current through it. We may consider this resistance in a way like mechanical friction. Less water will flow in a pipe that is rough than in a smooth pipe because friction cuts down the flow. Less electric current will flow in a wire or "conductor," as it is called, having a high resistance than in a low-resistance conductor. The effect of the electrical resistance is to cause a part of the electrical energy to be changed into heat. Every electrical conductor is therefore heated by an electric current flowing through it. The less the resistance of a conductor the less the heat produced by a given current. Electrical resistance is measured in ohms.

The resistance of a conductor depends on its material, that is, whether copper, aluminum or iron, upon its length, area of cross section and its temperature. Copper is a better conductor of electricity than either aluminum or iron, that is, it offers less resistance to the flow of the electric current through it. If copper wire is heated its resistance increases. The resistance of a wire increases as its length is increased and decreases as its area of

cross section is increased. In electric wiring it is necessary to have wire large enough in cross section to carry the current required without overheating. If a wire too small is used there is danger from fire and insurance rules limit the size of wire for different kinds of work. In calculating the resistance of wires of circular cross section, their area is often found in circular mils, by squaring the diameter expressed in thousandths of an inch. Thus a wire 14 in. or .25 in. in diam. has $(.25 \times 1000)^2$ or (62,500) circular mils in it. This is not the area as we determine the area of a circle, but is a convenient way of expressing relative areas of wire cables, etc. To find the resistance of copper wire at ordinary temperature we use the following rule:

Resis. in ohms $= \frac{10.8 \times \text{length of wire in ft.}}{\text{area of cross section in cir. miles}}$

Using this rule the resistance in ohms of 1 mile (5280 ft.) of copper wire .12 in. in diam, is equal to

$$\frac{10.8 \times 5280}{(.12 \times 1000)^2}$$
 or $\frac{10.8 \times 5280}{120 \times 120}$ 3.96 ohmo

For another example, the resistance of 2000 ft, of copper wire .20 in, in diam, is

$$10.8 \times 2000 \text{ or } 10.8 \times 2000 \text{ or .54 ohm}$$

 $(.20 \times 1000)^2 \text{ or } 40,000 \text{ or .54 ohm}$

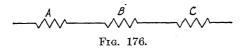
Ten miles of a 100,000 circular mil copper cable has a resistance of

$$\frac{10.8 \times 10 \times 5280}{100,000}$$
 or 5.70 ohm:

69. Resistances in Series. If a number of electrical conductors are arranged one after another in a current x in Fig 176 so that the same current flow through all of

them, they are said to be "in series" and their total resistance is equal to the sum of their separate resistances:

Total resistance = resistance of A + resis. of B+ resis. of C



70. Resistances in Parallel.—If the conductors are arranged side by side as in Fig. 177 so that the current divides in going through them and each conductor takes only a part of the total current, they are said to be "in parallel" and the reciprocal of the total resistance is equal to the sum of the reciprocals of the separate resistances:

$$\frac{1}{\text{Total resistance}} = \frac{1}{\text{resis. } A} + \frac{1}{\text{resis. } B} + \frac{1}{\text{resis. } C}$$

$$\frac{A}{C}$$
Fig. 177.

For example, if a copper cable of .10 ohm resistance is in parallel with an aluminum cable of .85 ohm resistance the combined resistance is found as follows:

or
$$\frac{1}{\text{total resistance}} = \frac{1}{.10} + \frac{1}{.85}$$

$$\frac{1}{\text{total resistance}} = 10 + 1.18 \text{ or } 11.18$$
and total resistance = $\frac{1}{11.18}$ or .0895 ohm.

or

For transmitting power, copper, aluminum, steel, and phosphor bronze wire are used, copper and aluminum principally. For telephone and telegraph work steel and iron wires are used. For protective devices such as fuses and for coils of electrical measuring instruments, various alloys or mixtures of metals are used.

Copper is used to a large extent as an electrical conductor on account of its comparatively low resistance. Aluminum has an advantage over copper in its lighter weight for the same conducting power. This in transmission lines reduces the weight on insulators, poles, and cross-arms.

PROBLEMS ON RESISTANCE

- 1. Find the resistance of 5 miles of transmission line with conductors .187 in. in diam.
- 2. It is desired to extend the above line 2 miles with conductors ,125 in, in diam. What is the resistance of the line added and of the total line?
- 3. Find the resistance of 12 miles of 150,000 circular mil copper cable.
- 4. An aluminum cable of 12 ohms resistance is put "in parallel" with the copper cable of Prob. 3. What is the resistance of the two in parallel?
- 5. A copper wire has a resistance of 20 ohms. Another wire of the same material is twice as long as the first and twice as large in cross section. How does its resistance compare with that of the first?
- 6. Give as many examples as you can of the use made of the heating effect of an electric current flowing through a resistance.
- 7. The resistance of a line wire is 4 ohms and a second wire of the same size is run along and "tied in" with the first, what is the resistance of the two taken together?
- 8. The resistance of a copper wire is 5 ohms. What is the resistance of another wire of the same material three times as long and of one-third the cross-sectional area of the first?

- 9. What metals are used as electrical conductors besides copper and aluminum? Explain in what kind of service they are used.
- 10. From the table on this page what number wire would you use for concealed work to carry a current of 22 amperes? What part of an inch is the diameter of this wire, and what does it weigh per mile if insulated?

71. Safe Carrying Capacity of Wires.—

Gauge No., B. & S.	Diameter, mils.	Ohms per 1000 ft.	No. amperes, open work	No. amperes, concealed work	Lb. per 1000 ft., bare	Lb. per 1000 ft., insulated
18	40	6.3880	5	3	4.92	18
17	45	5.0660	6	4	6.20	21
16	51	4.0176	8	6	7.82	25
15	57	3.1860	10	8	9.86	31
14	64	2.5266	16	12	12.44	38
13	72	2.0037	19	14	15.68	43
12	81	1.5890	23	17	19.77	48
11	91	1.2602	27	21	24.93	64
10	102	.99948	32	25	31.44	80
9	114	.79242	39	29	39.65	97
8	128	.62849	46	33	49.99	116
7	144	.49845	56	39	63.03	118
6	162	.39528	65	45	79.49	166
5	182	.31346	77	53	100.23	196
4	204	.24858	92	63	100 40	000
3	229	.19714	110	75	126.40	228
$\frac{3}{2}$	258	.15633	131	75 88	159.38	265
1	289	.12398	156	105	200.98	296
0	325	.09827	185		253.43	329
00	365	.07797	$\frac{185}{220}$	125 150	319.74	421
000	410	.06134	262	181	402.97	528
0000	460	.04904	312	218	508.12	643
	100	.04904	314	218	640.73	815

72. Ohm's Law, Calculation of Current, Voltage and Resistance.—In order to force an electric current through

a wire or conductor in spite of its resistance it is necessary to have an electric pressure just as it is necessary to have water pressure or "head" to force water through a pipe in spite of friction.

Electric pressure is measured in *volts* and is indicated by an instrument called a *voltmeter*. Instead of electric

pressure we sometimes speak of the "voltage" of a circuit or the "electromotive force" of the circuit. The three terms "electric pressure," "voltage," and "electromotive force" all mean the same pressure required to force an electric current through a circuit. The amount of current which will flow in a circuit of given resistance and under a given pressure is found

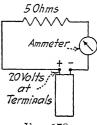


Fig. 178.

from the following rule known as Ohm's Law (named from Dr. Ohm):

$$Current = \frac{\text{voltage}}{\text{resistance}}$$

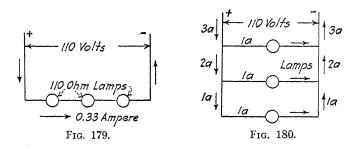
If the voltage is in "volts" and the resistance in "ohms" the current is in "amperes." For example, if, as in Fig. 178, a 20-volt battery is connected to a wire of 5 ohms resistance a current of $\frac{20}{5}$ or 4 amperes will flow through the circuit, and may be read on the "current meter" or "ammeter."

For example, if an incandescent carbon filament lamp having a resistance of 220 ohms is put on a 110-volt circuit, the current which it will take equals $\frac{110}{220}$ or .5 ampere. From the rule that

- (1) Current = $\frac{\text{voltage}}{\text{resistance}}$ it follows that
- (2) Voltage = current × resistance, and also
- (3) Resistance = $\frac{\text{voltage}}{\text{current}}$.

Rule (2) enables us to find the voltage across a known resistance through which a known current flows, and rule (3) enables us to find the resistance of a circuit with a known current flowing under a known pressure.

In Fig. 179, three lamps are arranged "in series" across 110-volt mains. Here the current through each



lamp is the same and if each lamp has a resistance of 110 ohms, the three "in series" have a resistance of 3×110 or 330 ohms. The current through the lamps therefore equals $\frac{\text{the voltage}}{\text{the resistance}}$, or $\frac{110}{330}$ or .33 ampere.

From rule (2) the voltage across each lamp equals current \times resistance, or .33 \times 110 or 36.7 volts.

Fig. 180 shows three 110-ohm lamps arranged "in parallel" across 110-volt mains. The current through each lamp equals $\frac{\text{the voltage}}{\text{resistance of lamp}} = \frac{110}{110}$ or 1 ampere. The three lamps together therefore take three amperes.

The arrows indicate the direction of flow of the current and the figures, the number of amperes in the different parts of the circuit.

PROBLEMS

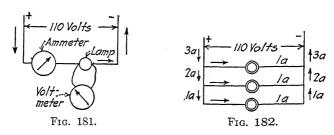
- 11. What is the resistance of a magnet winding through which 1.5 amperes of current flows under a pressure of 110 volts? Draw a figure showing the magnet in circuit?
- 12. If a 32-e.p. carbon lamp of 110 ohms resistance is connected across a 110-volt circuit, what current flows through it? How much current do ten of these lamps take when arranged "in parallel" on a 110-volt circuit?
- 13. How much current would each of ten 220-ohm lamps take if put "in series" in a 550-volt circuit?
- 14. A carbon filament lamp operating on 110 volts takes .95 ampere when first put in circuit and .93 ampere after burning for 10 min. What is the change in resistance due to heating the filament?
- **15.** Three 110-ohm lamps are connected in series and these in series with a *parallel* group of five 220-ohm resistances. What is the resistance of the total number of lamps as arranged?
- 16. In Prob. 15 what is the voltage across each part of the circuit when 110 volts are impressed on the total circuit described?
- 17. Two thousand feet of No. 18 B. & S. gauge copper wire is to be wound on a magnet and operated on a 110-volt circuit. What resistance must be put "in series" with this winding in order that the current may be kept within safe limits? (See table, page 194.)
- 18. What is the weight of 10 miles of No. 00 B. & S. gauge trolley wire?
- 19. What is the total current taken by ten lamps in parallel if each has a resistance of 121 ohms and operates on a 110-volt circuit?
- 73. Calculation of Power and Energy in an Electric Circuit. The power consumed in a direct current circuit and the power taken by an incandescent lamp, whether on a direct or alternating current circuit equals the product

of the current flowing through the circuit and the voltage across the circuit, that is,

$$power = current \times voltage$$

When the current is in "amperes" and the voltage in "volts," the *power* is in watts.

For example, if an incandescent lamp, as in Fig. 181, across 110-volt mains and has 1 ampere flowing



through it, the *power* in watts equals the voltage 110 multiplied by the current 1 or watts = 110×1 or 110.

In Fig. 182, three 110-ohm lamps are shown in parallel across 110-volt mains. By the rule for finding the current, we have the current taken by each lamp =

$$\frac{\text{the voltage across lamp}}{\text{resistance of the lamp}}$$
 or $\frac{110}{110} = 1$ ampere.

The power taken by each lamp = current through lamp \times voltage across lamp = 1×110 or 110 watts. The three lamps together therefore take 110×3 or 330 watts. The arrows and numbers in the figure show the direction of flow of the current and the number of amperes in each part of the circuit.

One thousand watts is called a kilowatt. In the

example above 330 watts or $\frac{330}{1000} = .330$ of a kw. is the *power* consumed constantly.

The energy consumed in an electric circuit equals the product of the power and the time during which the power is taken. If the power is in watts and the time in hours, their product gives the energy in watt-hours. If the power is in kilowatts and the time in hours their product gives the energy in kilowatt-hours which is the unit used by electric light and power companies. If lights which take .30 kw. are kept burning for 3 hr. the energy they consume in this time equals kilowatts \times hours or .30 \times 3 or .90 of a kw.-hr.

Electrical energy is sold at prices ranging from a fraction of a cent to 18 or 20 cts. per kw.-hr. (kilowatthour). At 8 cts. per kw.-hr. the cost of the energy consumed by the lamps taking .90 kw.-hr. is .90 \times 8 or 7.2 cts.

PROBLEMS

- 20. What power is taken by a 110-volt carbon filament lamp with a current of .98 ampere?
- 21. A tungsten lamp takes .545 ampere at 110 volts. What is its wattage?
- 22. What is the difference in power taken by a group of five 110-volt carbon filament lamps each taking 1 ampere and a cluster of three 110-volt tungsten lamps each taking 60 watts?
- 23. Calculate the kilowatt-hours of energy consumed in 5 hr. by three 100-watt tungsten lamps.
- 24. How much energy is taken in 2 hr. by twelve 60-watt lamps?
- 25. What is the cost per month of 30 days of 2 hr. of lighting per day, for an entire building with energy of 8 cts. per kw.-hr., if there are twelve 100-watt lamps and the equivalent of twenty 40-watt lamps in the building?
 - 26. Find the cost per month (30 days) for house lighting, with

4 carbon filament lamps each taking 1 ampere at 110 volts and burning for an average of 5 hr. per day. Take the cost of energy at 8 cts. per kw.-hr.

27. What kinds of incandescent lamps are in general use? State the advantages and disadvantages of each.

- 28. An office is first lighted with six 16-c.p. carbon filament incandescent lamps taking 3.1 watts per candle. The carbon lamps are later replaced by four 60-watt tungsten lamps. What is the saving per month of 30 days with the lamps burning 5 hr. per day if energy costs 8 cts. per kw.-hr.?
- 29. How does the first cost of carbon and tungsten lamps compare? Is there a saving in using tungsten rather than carbon lamps?
- 74. Electrical Horsepower.—Horsepower means the rate at which work is done. For example, 33,000 ft.-lb. of work done per min. or 550 ft.-lb. per sec. represents one horsepower. In electrical terms one horsepower (abbreviated h.p. or HP) equals 746 watts. To find the horsepower taken by a direct-current motor multiply the voltage on the motor by the current taken by the motor and divide the product by 746, that is

 $horsepower of a motor = \frac{Volts on motor \times current intake}{746}$

Example.—Find the horsepower consumed by a 110-volt motor taking a total current of 10.3 amperes.

Horsepower =
$$\frac{110 \times 10.3}{746}$$
 or 1.52 Ans.

To find the horsepower delivered by an electric generator, multiply the voltage at the generator terminals by the current output and divide the product by 746, that is

horsepower of a generator $=\frac{\text{Volts at gen. ter.} \times \text{curr. out.}}{746}$

Example.—An electric generator with 110 volts at its terminals supplies 25 amperes to a lamp board. What horsepower is it giving out as useful power?

Horsepower =
$$\frac{110 \times 25}{746}$$
 or 3.69 Ans.

To find the horsepower-hours, multiply the load in horsepower by the time in hours during which the power is supplied.

Example.—A generator supplies 13 amperes at 120 volts for 5 hr. what is its horsepower hour output?

Horsepower hours =
$$\frac{120 \times 13}{746} \times 5 = 10.45$$
 Ans.

PROBLEMS

30. What horsepower generator is necessary to supply 30 amperes at a pressure of 110 volts?

31. A motor driving a lathe takes 6.4 amperes at 220 volts, what is the horsepower supplied to the motor?

- 32. Three direct-current and voltage ratings are as follows: 10 amperes, 110 volts; 15 amperes, 220 volts; 30.5 amperes at 110 volts. Find the corresponding horsepower rating of each of these motors when working at full load.
- 33. Five tungsten lamps, each take .95 ampere at 110 volts, what is the number of watts and horsepower taken by the lamps? If the lamps burn for 5 hr. what energy do they take in watt-hours, kilowatt-hours, and in horsepower-hours?
- 34. What kind of instruments are used for measuring an electric current? Find out over what range these instruments are made to read.
- **35.** What kind of instruments are used for measuring electric pressure or voltage? Find out what range these instruments have.
 - **36.** How are electrical "power" and "energy" measured?
- 37. Find out what different kinds of electric lamps are in general use, with the advantages and disadvantages of each.
- 38. What rules do you know in regard to personal safety and protection of machines when working about electrical circuits?

75. Methods of Charging for Electrical Energy.—
(a) FLAT RATE SYSTEM.—In the flat rate system the customer is charged a certain amount per month for each lamp installed. No meters are used to determine the amount of energy consumed. With this system a waste-

amount of energy consumed. With this system a wasteful consumer pays no more for the same number of lights installed than an economical user. This system saves the cost of meters, their repair, and simplifies the com-

pany's bookkeeping. A scale of discounts is usually arranged increasing with the number of lamps installed.

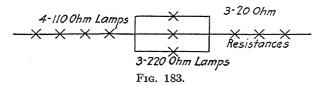
(b) METER SYSTEM.—In this system the consumer is charged for the energy which he uses, that is, for the number of kilowatt-hours which he consumes in a certain time. This method would seem to be a just way of charging for energy, but the time of day that power is taken is of as much importance as the amount taken. Power taken at a time when the station load is heavy is more expensive for the company than power taken at a time of light load. The cost of power depends on the size of the power station, the amount of power taken, and the time at which it is taken. In view of these facts a "two rate" meter is sometimes used with two sets of dials, one reading at times of light station load and the other at times of heavy station load. The consumer is then charged one rate for the power taken when the station is working light and a higher rate for power taken at times of heavy load on the station.

A "maximum rate" meter is also used in some cases. This meter gives the greatest load or power which the consumer has taken during a certain period. The charge is then made on a basis of the greatest amount of power taken at any one time as well as on the total number of kilowatt-hours consumed.

A certain charge is usually made whether the customer uses any power or not. This is because the customer has power at hand any time he wishes to use it, and a charge is made for this service.

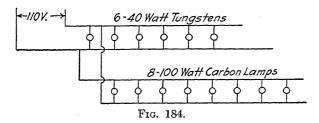
PROBLEMS

- 39. Find the total resistance of the circuit shown in Fig. 183.
- 40. If the circuit of Prob. 39 is connected across 110-volt mains, what is the current through the circuit and the voltage across each portion of the circuit?



- 41. Find the difference in power taken by eight 220 ohm lamps operating on a 220-volt circuit and sixteen 100-watt lamps.
- 42. If a current of 8.2 amperes flows through a resistance on 110 volts, how much more resistance must be put in circuit to reduce the current to 4.5 amperes?
- **43.** How many horsepower-hours are equivalent to 2000 kw.-hr.? How many kilowatt-hours are consumed by a lamp load of 125 amperes at 220 volts in circuit for 4 hr.?
- 44. Calculate the power in watts taken by thirty 110-volt carbon lamps connected in parallel if each lamp takes 1.5 amperes.
- 45. Calculate the power taken by 20 lamps in series on a 1100-volt circuit if 15 amperes flow through the circuit.
- 46. Calculate the power in kilowatts taken by ten 110-volt lamps in parallel across a 110-volt circuit if each takes .9 ampere.
- 47. Ten lamps taking 1 ampere each are connected in parallel across 110-volt mains. What is the resistance of each lamp and the power taken by the circuit?
- **48.** Four 110-volt carbon lamps taking 1 ampere each were replaced by four 40-watt tungsten lamps. If the cost of energy was 8 cts. per kw.-hr., how much did they gain or lose per hour by the change?

- 49. Find the difference in kilowatt-hours taken in 5 hr. by ten 220-ohm lamps on 110-volts and eight 60-watt tungstens.
- 50. Find cost of operating the circuit shown in Fig. 184 per month of 30 days and 4 hr. per day, energy 10 cts. per kw.-hr.
- 51. What system of charging for electricity do you consider the best, and why?



76. Comparison of Water and Electric Systems.—A very close and interesting comparison can be made between a water system with its piping, gauges, pumps, motors and so forth and an electric system with its wiring, meters, generators, motors and so forth. In Figs. 186 and 187 diagrams are shown of typical water and electric systems, with the corresponding parts in the two systems marked and explained as follows:

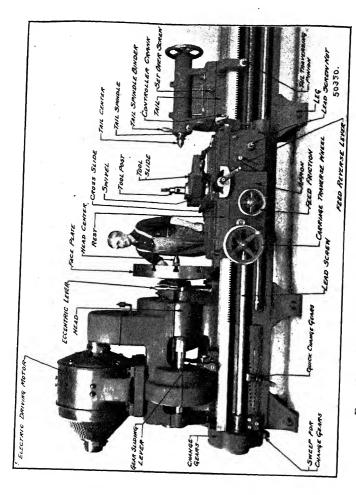


Fig. 185.—Cut showing a lathe with an electric motor drive.

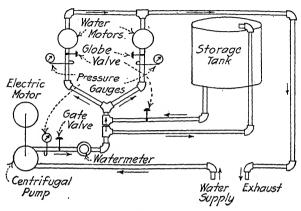


Fig. 186.

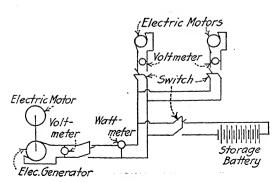


Fig. 187.

Water system	Corresponding part in electric system		
Electric motor for driving centrifugal pump.	Electric motor for driving electric generator.		
Centrifugal pump.	Electric generator.		
Pressure gauge, giving water pressure.	Voltmeter giving electric pressure.		
Gate valve for shutting off water.	Switch for opening electric circuit.		
Water meter for measuring quantity of flow of water.	Wattmeter for measuring the power taken by the circuit.		
Globe valve for shutting off water supply to motor.	Switch for cutting off electric current to electric motor.		
Water motor for doing mechanical work.	Electric motor for doing me- chanical work.		
Storage tank for water pumped by centrifugal pump.	Storage battery for electricity supplied by generator.		

CHAPTER XX

STRENGTH OF MATERIALS

77. Definition.—All materials have a property or characteristic which we call "strength," which is quite different for different materials such for example as wood and steel. In order to know what kind of material to use for a particular piece of work and how much of it to use, that is how large a piece is required in order that the completed work or job shall be sufficiently strong to serve its purpose, it is necessary to know what the strength of the various materials is and how this strength differs for the different methods in which pressure may be brought on the material.

The average person is likely to speak only relatively regarding the strength of materials, namely that one material is so many times stronger than another. It is necessary, however, in working with machines and in general construction work to have more definite knowledge of the strength of materials. This has led to the construction of testing machines, capable of measuring directly in pounds the strength of the materials under various conditions. These machines are described later on in this chapter.

The object of this chapter is to convey some idea of the strength of materials, from results secured by actual tests and how these results are used in calculating the size and amount of material required. This chapter also brings out some of the effects of forces applied to materials in changing their size and shape, and also in merely producing pressure in them without changing their form.

78. Stress and Strain.—Whenever a pressure or a load is put upon any piece of material it tends to change the shape of the piece. The material itself resists this change of shape and in so doing exerts a force opposite to the load or pressure applied to the material. For example, if we take a piece of string and, holding it in our hands, pull on it with an increasing force the string will finally break, but not until it has straightened out and become taut. After this there is a gradual stretch until the fibers pull apart. Our effort or pull on the string has been opposed by a resistance in the fibers of the string. This resistance is called stress. Stress may therefore be defined as the resistance which a material offers to the action of an external force, pressure or load which tends to change the shape of the material.

If a 2-ton weight is being lifted by a rope the stress produced in the rope is 4000 lb.

The stretching of the string lengthened or *elongated* it and produced a change of its form. This change of form in the fibers of a material is called *strain*.

In construction work the materials used must be of sufficient size and strength so that the loads applied are not sufficient to change their shape, otherwise the parts are likely to break or "rupture" and disastrous results follow.

- 79. Kinds of Stresses.—There are three general kinds of stresses, namely (1) Tensile stress (or pulling stress),
- (2) Compressive stress (pushing or crushing stress),
- (3) Shearing stress (cutting stress).

There are two other kinds of stresses which in reality are combinations of those given above. These are (a) Torsion or twisting which is a kind of shearing stress

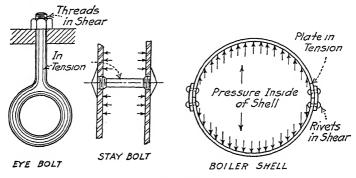
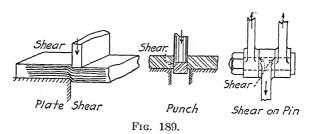


Fig. 188.

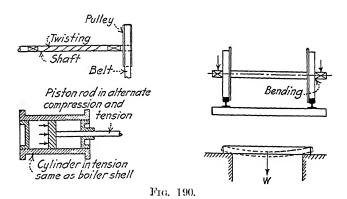
and (b) Bending or flexure which is a combination of tension and compression with or without shearing. The examples (Figs. 188, 189 and 190) show the various stresses of materials in service.



Since the tensile stress set up in a material is due to a pull, the *tensile strength* of the material is a measure of the resistance offered by its fibers to being pulled apart.

Since the compressive stress is a stress set up in a material due to a push, thrust, or crushing force, the *compressive strength* is a measure of the resistance offered by the fibers of the material to being crushed.

A shearing stress is one set up in a material due to a force acting at right angles to the material and the shearing strength is a measure of the resistance of the fibers of the material to being cut or sheared off.



80. Ultimate Strength.—If samples of different materials are loaded until they break or rupture we can find out how much each material will stand. The ultimate strength of a material is the number of pounds per square inch of cross section of the material at which it breaks. This value varies between wide limits for different materials and often varies considerably for the same material.

The ultimate strength of a given material is different for the different stresses of tension, compression and shear. 81. Testing Machines.—For testing materials to find their ultimate strengths, testing machines as shown in the following figures are used.

Fig 191 shows a machine used for tension and com-

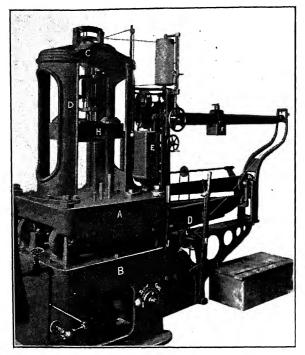


Fig. 191.

pression tests. The figure shows a piece being tested in tension.

The test specimen is placed in the machine as shown between heads (H) and (C) and is held firmly in place by means of wedge grips. Head (C) is fixed to the body

of the machine (A) by frame (D). The bed (A) rests on a set of levers. Through the head (H), two long screws are threaded which are geared to bevel gears shown inside of the case (B). A belt connected to a pulley which is fastened on a shaft to which the gears are fastened, drives the machine, which causes the screws to revolve slowly. This action moves (H) down causing a pull on the test piece and hence on the head (C), the pull being transmitted to (A).

(A) resting on lever (D) forces (D) down and by the system of levers, causes the lever on which (F) rests to move up. The system of levers is kept balanced by

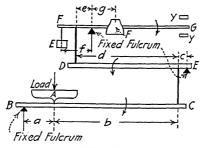


Fig. 192.

moving the weight (F) out along the graduated bar. (E) and (M) are weights used for balancing and for adjustment. All readings of the load applied and also the breaking load, are registered by means of (F), the lever on which it moves being marked off in pounds so as to read directly the pull exerted on the test piece.

When a piece is to be tested in compression, it is placed between the head (H) and (A) and it can be seen that as the head (H) moves down, the piece is compressed and the load registered as before.

The system of levers is shown in Fig. 192 giving an outline of their arrangement.

As the load is applied to the test specimen, it pushes down on (A) and tends to move lever (BC) around the fixed point at (B). The force at (C) then pulls down on the lever (DE) which is pivoted at (E). The lever (FG) is connected to the lever (DE) by the link (FD),

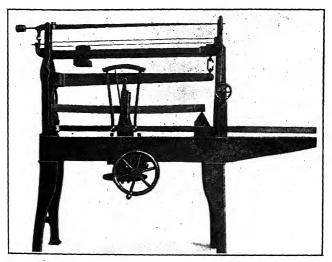
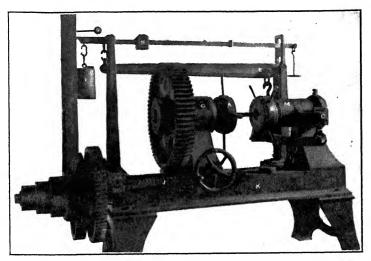


Fig. 193.

and as the force at (D) pulls down on the end (F) of lever (FG), the end (G) tends to move up. The end (G) is kept balanced between (Y) and (Y) by moving the weight (F) out along the lever. The object of the system of levers is merely to reduce the large load acting at (A) to a small load acting at (F). Nearly all testing machines are built on the principle of a system

of levers and the methods are worked out in a manner practically similar to that just described.

For bending tests of wood specimens a machine similar to that shown in Fig. 193 is used. The lever arrangement is similar to that of the tension machine. The method of supporting the specimen and recording the load on the beam and the deflection of the beam can be readily seen. This machine is operated by hand.



Frg. 194.

Pieces to be tested in torsion are placed in a machine similar to that shown in Fig. 194. The specimen is fastened in grips (A) and (B). (B) is fastened to (M) which has an arm (C) attached to it. By means of the belt shown at the left and the cone pulley and system of gears, (A) can be rotated. As it moves it tends to turn (B) and (M) and (C) tends to rise. Arm (C) then

pulls up on (D) which is a lever pivoted to the frame and so that the force is carried to the lever on which (H) rides. The lever is kept balanced by moving weight (H). The weight (F) serves as a balance. The system of levers here shown is practically the same as for the machine shown in Fig. 193 and the method of recording the load is similar.

82. Standard Test Pieces.—Figs. 195, 196 and 197 show a few of the standard test pieces or specimens used in finding the strength of materials.

The specimen used should in any case be as near an average specimen of the material as possible. If a

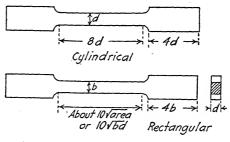


Fig. 195.

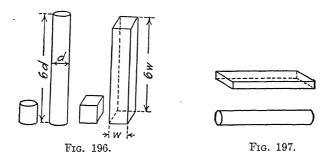
shipment of material is to be purchased, a few samples should be taken from different parts of the supply.

Fig. 195 shows a cylindrical and rectangular section of iron or steel as used in a *tension* test. The pieces are machined all over and filed along the middle part, for the body of the material should be as free from tool marks as possible and of a uniform size. The proportion of the parts is given in terms of the cross section of the material.

The blocks shown in Fig. 196 represent about the

limit of the size of pieces used in compression tests. A piece should not be longer than 6 times its diameter, else a bending may occur along with the compression.

The forms used for bending and shearing tests are like those shown in Fig. 197, the form in each case depending on the kind of material and the result to be determined. For torsion tests, the specimens are like those used for tension tests.



83. Values of Ultimate Strengths.—From the results of tests with these machines the following table shows the average values of ultimate strengths of the most used materials and for the kind of stress indicated.

ULTIMATE STRENGTH IN POUNDS PER SQUARE INCH

Material	Tension ,	Compression	Shear
Cast iron Wrought iron Average steel Soft copper Hard copper Wood	20,000 55,000 60,000 25,000 40,000	90,000 50,000 70,000 40,000 55,000 8,000	20,000 40,000 50,000 20,000 40,000 3,000 (across grain)

9			
Material	Tension	Compressor	Shear
Granite		15,000	1,500
Limestone			1,400
Concrete		2,000	1,000
Brick		5,000	1,000

The following results represent a fair average

It would be of course unsafe to use in actual service material under such conditions that it would be subjected to a stress near the ultimate stress of the material, since the piece might break at any moment. A piece of material put in actual service is therefore designed for a stress per square inch, which is much less than the ultimate stress. This stress is called the working stress or the safe working stress.

- 84. Factor of Safety.—The ultimate strength divided by the working stress is called the Factor of Safety. For example if a piece of steel breaks at 80,000 lb. per sq. in. and we use a factor of safety of 8, the working stress is only ½ of 80,000 lb. or 10,000 lb. per sq. in., and the design is worked out for the latter value. The factor of safety of a material varies according to the material used and the manner of loading. A much larger factor of safety is allowed for bridge members on which there is a varying load than for a building for example where the load is practically constant.
- 85. The Elastic Limit.—In designing structures such values of working stress are always taken that the material will not be stretched beyond its "elastic limit," that is will not be worked at a unit stress beyond which the material will not return to its original shape when the stress is removed. The Elastic Limit which should always be greater than the working stress may also be defined as that unit stress beyond which the deforma-

tions of the material increase in a faster ratio than the applied loads, and beyond which the material will acquire a permanent set on the removal of the load, and not return to its original shape.

86. Values of Safe Working Stresses.—Following are safe working stresses in pounds per square inch for the most commonly used materials. The safe working load for tension, compression and shear are abbreviated respectively as follows S_t , S_c , S_s .

SAFE WORKING STRESSES-POUNDS PER SQUARE INCH

Material	$\begin{array}{c} \text{Tension} \\ \mathcal{S}_{\boldsymbol{\ell}} \end{array}$	Compression Se	$_{S_s}^{\rm Shear}$
Cast iron	4,000 10,000 12,000 800	12,000 12,000 16,000 800	3,000 10,000 12,000 600 (with grain)

From the above we have the rule, the safe load "W" which can be carried for the several kinds of stresses is as follows:

For tension
$$W = \text{area in sq. in.} \times S_t$$
.
For compression $W = \text{area in sq. in.} \times S_c$.
For shear $W = \text{area in sq. in.} \times S_s$.

Or in general for all stresses

$$W = \text{area} \times S$$
 from which
$$\text{area} = \frac{W}{S}$$

These mean that the total safe load equals the stress per square inch times the area of cross section of the piece in square inches. And also the area required by a piece equals the total load divided by the safe load per square inch.

From the calculated area, the length of a side of the cross section, or the diameter in case of a circle, and so forth may be found.

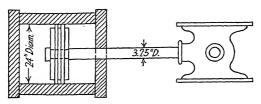
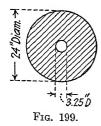


Fig. 198.

87. Strength of Rods.—When steam is applied to the back side of the piston shown in Fig. 198, the pressure puts the rod in tension. If the pressure is 180 lb. per sq. in. the total pressure on the piston is 180 times the net area of the piston, that is, subtracting the area of



cross section of the rod. The area on which the pressure acts is shown cross sectioned in Fig. 199 and equals

$$(24)^2 \times .785 - (3.25)^2 \times .785$$
 or 444.1 sq. in.

The total pull on the rod therefore equals

$$444.1 \times 180 \text{ or } 79,900 \text{ lb.}$$

that is, the applied load is 79,900 lb. and the total resisting stress is therefore 79,900 lb. The stress per sq. in. on the rod equals

$$\frac{\text{total load}}{\text{area of rod}}$$
 or $\frac{79,900}{8.30}$

which equals 9630 lb., and this is well within the safe limit for steel. The unit compressive stress is also within the safe limit for steel. The compressive stress occurs when steam is admitted to the head end of the cylinder.

Example.—Find the size of a circular steel rod from which a weight of 4 tons is to be suspended.

Solution.—Taking 14,000 lb. per sq. in. as the safe working load in tension, we have

Area =
$$\frac{W}{S}$$
 or $\frac{4 \times 2000}{14,000}$ or .572 sq. in.

The diameter of the rod then equals

$$\sqrt{\frac{\text{area}}{.785}}$$
 or $\sqrt{\frac{.572}{.785}}$ or .853 in.

In figuring out threaded bolts the required area must be taken as that at the *root* of the threads, since a piece is no stronger than its weakest part. If the bolt has a sufficient strength for the area at the root of its thread, or in other words for its *smallest* area, the body of the bolt will have a surplus of strength on account of its greater area of cross section.

88. Strength of Ropes, Chains and Cables.—The ultimate strength of hemp rope is about 6000 lb. per sq. in. and for Manila rope about 3000 lb. per sq. in.

A safe working stress for hemp rope is 1400 lb. per sq. in. based on the *Nominal Area* of the rope, that is, assuming the area of cross section of the rope a solid circle. For Manila rope half the above value may be used.

The safe working load for chain links is given as 9000 lb. per sq. in. For iron cables the ultimate strength is about 40,000 lb. per sq. in. and for steel cables 80,000

1

lb. per sq. in. A factor of safety of 10 is often used for ropes and cables in finding the safe working stresses. This makes the safe value of working stress for iron cable 4000 lb. per sq. in. and for steel cable 8000 lb. per sq. in.

89. The Strength of Columns.—When a bar or rod subjected to compression has a length greater than 10 times its diameter it is called a column, and it must be worked out by difficult formulas which take account of its length, size and form of cross section. Columns may fail by buckling as well as being crushed, and most satisfactory results in some cases are obtained by testing the actual columns in a suitable testing machine. From the results of such tests the most reliable information on the strength of the columns is obtained.

PROBLEMS

1. It took 100,000 lb. pull in a testing machine to break a steel rod $1\frac{1}{2}$ in. in diam. What was its ultimate tensional strength in lb. per sq. in.?

2. If a cast-iron bar $1\frac{1}{2}$ in. by 2 in. in cross section breaks under a tensile load of 70,000 lb., what load in lb. per sq. in. would break a $2\frac{1}{2}$ -in. diam. cast-iron rod of equal unit strength?

- 3. A round cast-iron bar is to be subjected to a tension of 34,000 lb. If it is designed so that the unit stress is 2500 lb. per sq. in., find the diameter of the rod.
- 4. A wooden block 15% in. square was crushed in a testing machine under a load of 21,640 lb. What was its ultimate compressive strength in lb. per sq. in.?
- 5. It required 87,630 lb. to shear off in a testing machine a $1\frac{1}{4}$ -in. diam. steel rod. What was its ultimate shearing strength in lb. per sq. in.?
- 6. If the ultimate compressive stress of wrought iron is 55,000 lb. per sq. in. and it is subjected to a working stress of 11,500 lb. per sq. in., what is its factor of safety?
- 7. A 7/s-in. diam. wrought-iron tie rod on a bridge is subjected to a working stress of 9000 lb. per sq. in. What load is it sup-

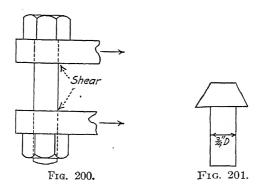
porting? If the ultimate stress is 50,000 lb. per sq. in., what factor of safety is used in its design?

8. Find the compression in lb. per sq. in. for a 3-in. diam. piston rod carrying an 18-in. diam. piston with a steam pressure of 180 lb. per sq. in.

9. Find the tensile stress in lb. per sq. in. for a 3½-in. diam. piston rod with steam at 175 lb. per sq. in. acting on the back of a 20-in. diam. piston attached to the rod.

(Note.—Subtract the cross-sectional area of the piston rod from the piston area to get the *net* area on which the steam acts.)

10. What is the stress per sq. in. in the studs holding on a cylinder head for a 24-in. diam. cylinder if there are twenty-six



 \mathcal{V}_8 -in. diam. studs and the greatest steam pressure is 200 lb. per sq. in?

(Note.—The area at root of thread of each 1/8-in. stud is .419 sq. in.)

11. A solid east-iron cylinder 2½ in. in diam. is under a compression of 40,000 lb. Find its factor of safety.

12. A brick 2 in. \times 4 in. \times 8 in. weighs about $4\frac{1}{2}$ lb. Find the height of a pile of bricks so that the compressive stress per sq. in. on the lowest brick will be $\frac{1}{2}$ of the ultimate strength.

13. A wrought-iron bolt as shown in Fig. 200 is to be subjected to shear in two places. If the load is 3000 lb. and the factor of safety 5, find the diameter of the bolt.

- 14. What size of hemp rope is required to lift a casting weighing 21/2 tons?
- 15. What is the total safe load which a steel cable 2 in. in diam. can stand?
- 16. For an ultimate tensile strength of 50,000 lb. per sq. in. find the strength of the 34-in. diam. cone rivet shown in Fig. 201.

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